2015 WASSCE FURTHER MATHEMATICS / MATHEMATICS 2 THEORY

SECTION A

1. A binary operation Δ is defined on the set of real numbers, R, by a Δ b = $a^3 - b^3$. Without using a calculator, find the value of (√3 + √2) Δ (√3 − √2), leaving the answer in surd form.

2. Points (2, 1) and (6, 7) are opposite vertices of a square which is inscribed in a circle. Find the: (a) centre of the circle; (b) equation of the circle.

3. If f''(x) = 2, f'(1) = 0 and f(0) = −8, find f(x).

4. Solve: tan (2x − 15°) − 1 = 0, for values of x such that 0° ≤ x ≤ 360°.

5. A car moving with an initial velocity, u, travels in a straight line with a constant acceleration of 3 ms^{-2} until it attains a velocity of 33 ms^{-1} after 6 seconds. Calculate the distance travelled by the car.

6. Five finalists in a beauty pageant were ranked by two judges X and Y as shown in the table.

Judges	Anne	Linda	Susan	Rose	Erica
X	1	4	3	5	2
Y	3	2	4	5	1

Calculate the Spearman's rank correlation coefficient.

7. There are 8 boys and 6 girls in a class. If two students are selected at random from the class, find the probability that they are of: (a) the same sex; (b) different sex.

8. Forces of magnitude 3 N, 4 N and 2 N act along the vectors j, −i + j and i + j respectively. Calculate, correct to one decimal place, the magnitude of the resultant of the forces.

SECTION B

PART I: PURE MATHEMATICS

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9. (a) The functions f: $x \rightarrow x^2 + 1$ and g: $x \rightarrow 5 - 3x$ are defined on the set of real numbers, R.

(i) State the domain of f^{-1}, the inverse of f

(ii) Find $g^{-1}(2)$

(b) Evaluate $\int_0^2 (x + 3)dx / \sqrt{(x^2 + 6x + 9)}$

10. (a) If $(\sqrt{5} + 4)/(3 - 2\sqrt{5}) - (2 + \sqrt{5})/(4 - 2\sqrt{5}) = a + b\sqrt{5}$, find the values of a and b.

(b) (i) Evaluate:

2	**−1**	**2**
1	3	4
1	2	1

(ii) Using the result in 10(b)(i), find, correct to two decimal places the value of x in the system of equations:

$2x - y + 2z + 5 = 0$

$x + 3y + 4z - 1 = 0$

$x + 2y + z + 2 = 0$

11. (a) (i) Write down the Binomial expansion of $(1 + x)^4$.

(ii) Use the result in 11(a)(i) to evaluate, correct to three decimal places $(5/4)^4$.

(b) The first, second and fifth terms of a linear sequence (A.P.) are three consecutive terms of an exponential sequence (G.P.). If the first term of the linear sequence is 7, find its common difference.

PART II: STATISTICS AND PROBABILITY

12. The table shows the distribution of the heights of a group of people.

Height/m	**0.4–0.5**	**0.6–0.9**	**1.0–1.2**	**1.3–1.4**	**1.5–1.7**
Number of People	2	8	12	6	6

(a) Draw a histogram to illustrate the distribution. SEE DIAGRAM

(b) Using an assumed mean of 1.1 m, find, correct to one decimal place, the mean height of the group.

13. (a) Edem and his wife were invited to a dinner by a family of 5. They all sat in such a way that Edem sat next to his wife. Find the number of ways of seating them in a row.

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(b) A bag contains 4 red and 5 black identical balls. If 5 balls are selected at random from the bag, one after the other with replacement, find the probability that: (i) a red ball was picked 3 times; (ii) a black ball was picked at most 2 times.

PART III: VECTORS AND MECHANICS

14. (a) Given that AB = (4, 5) and BC = (−3, 5) (column vectors), find the:
(i) angle between the vectors AB and AC;
(ii) unit vector along AB − BC.

(b) P, Q, R and M are points in the OXY plane. If PQ = 2i + 8j, PR = 11i − 12j and M divides QR internally in the ratio 3 : 7, find PM.

15. (a) A bucket full of water with a mass of 8 kg is pulled out of a well with a light inextensible rope. Find its acceleration when the tension in the rope is 150 N. [Take g = 10 ms^{-2}]

(b) A mass of 12 kg is acted upon by a force F, changing its speed from 15 ms^{-1} to 25 ms^{-1} after covering a distance of 50 m. Find the: (i) value of F; (ii) distance covered when its speed is 35 ms^{-1}.

(b) A bag contains 4 red and 5 black identical balls. If 5 balls are selected at random from the bag, one after the other with replacement, find the probability that: (i) a red ball was picked 3 times; (ii) a black ball was picked at most 2 times.

PART III: VECTORS AND MECHANICS

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(b) A mass of 12 kg is acted upon by a force F, changing its speed from 15 ms^{-1} to 25 ms^{-1} after covering a distance of 50 m. Find the: (i) value of F; (ii) distance covered when its speed is 35 ms^{-1}.

2016 WASSCE FURTHER MATHEMATICS / MATHEMATICS 2 THEORY

SECTION A

Answer all the questions in this section

1. A binary operation * is defined on the set, R, of real numbers by m * n = m + n + 2. Find the: (a) identity element under the operation *; (b) inverse of n under the operation *.

2. Given that (5, 2), (−4, k) and (2, 1) lie on a straight line, find the value of k.

3. (a) If $f(x + 2) = 6x^2 + 5x - 8$, find f(5).
(b) Express $(7\sqrt{2} + 3\sqrt{3})/(4\sqrt{2} - 2\sqrt{3})$ in the form $p + q\sqrt{r}$, where p, q and r are rational numbers.

4. When $f(x) = 2x^3 + mx^2 + nx + 11$ is divided by $x^2 + 5x + 1$, the quotient is $2x - 5$ and the remainder is $30x + 16$. Find the values of m and n.

5. The probabilities that Ago, Sulley and Musa will gain admission to a certain university are 4/5, 3/4 and 2/3, respectively. Find the probability that: (a) none of them will gain admission; (b) only Ago and Sulley will gain admission.

6. The mean of the numbers 1, 4, k, (k + 4) and 11 is (k + 1). Calculate the: (a) value of k; (b) standard deviation.

7. (a) A body of mass 3 kg moves with a velocity of 8 ms^{-1}. It collides with a second body moving in the same direction with a velocity of 5 ms^{-1}. After collision the bodies move together with a velocity of 6 ms^{-1}. Find the mass of the second body.

(b) If the second body in 7(a) moves with a velocity of 5 m/s, in the opposite direction as that of the 3 kg body with a velocity of 8 m/s, find, correct to 2 decimal places, the common velocity of the two bodies if they move together after collision. (handwritten on the page)

8. Given that p = (5, 3), q = (−1, 2), r = (17, 5) (column vectors) and r = αp + βq, where α and β are scalars, express q in terms of r and p.

SECTION B

Answer four questions only from this section

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9. (a) Without using mathematical tables or calculator, evaluate [(3/2) log 27 − 3 log 5√5] / log 0.6

(b) Two linear transformations A and B in the Oxy plane, are defined by

A: $(x, y) \rightarrow (x + 2y, -x + y)$

B: $(x, y) \rightarrow (2x + 3y, x + 2y)$

(i) Write down the matrices A and B.

(ii) Find the image of the point P(−2, 2) under the linear transformation A followed by B.

10. (a) (i) Write down the expansion of $(1 + x)^7$ in ascending powers of x.

(ii) If the coefficients of the fifth, sixth and seventh terms in the expansion in 10(a)(i) form a linear sequence (A.P.), find the common difference of the A.P.

(b) Using the trapezium rule with ordinates at 1, 2, 3, 4 and 5, calculate, correct to two decimal places, $\int_1^5 \sqrt{(2x + 8x^2)}\, dx$.

11. (a) If ${}^kP_2 = 72$, find the value of k.

(b) Solve the equation $2\cos^2\theta - 5\cos\theta = 3$, for $0° \leq \theta \leq 360°$.

PART II

12. The table shows the distribution of marks scored by some students in a test.

Marks	**1–10**	**11–20**	**21–30**	**31–40**	**41–50**	**51–60**	**61–70**	**71–80**	**81–90**	**91–100**
Number of students	3	17	41	85	97	115	101	64	21	6

(a) (i) Construct a cumulative frequency table for the distribution.

(ii) Draw a cumulative frequency curve for the distribution. SEE DIAGRAM

(b) Use the curve to estimate the:

(i) number of students who scored marks between 32 and 74;

(ii) pass mark, if 18% of the students failed;

(iii) lowest mark for distinction, if 8% of the students passed with distinction.

13. (a) Two different Mathematics books, 5 different Physics books and 3 different Chemistry books are to be arranged on a shelf. How many

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arrangements are possible if:

(i) books on the same subject must stand together?

(ii) only the Physics books must stand together?

(b) In a certain community, 13 out of every 20 persons speak English. If 8 persons are selected at random from the community, find, correct to three significant figures, the probability that at least 3 of them speak English.

PART III

14. Four vectors $r = \alpha i + \beta j$ where α and β are positive constants, $s = 2i - j$, $m = 3i + 2j$ and $n = i + j$ are such that the magnitude of r is three times as s and is parallel to the vector $(m - n)$.

(a) Find the values of α and β.

(b) Calculate the magnitude and direction of $(r - s)$.

15. A particle is projected vertically upward from the ground with speed 30 ms^{-1}. Calculate the:

(a) maximum height reached by the particle;

(b) time taken by the particle to return to the ground;

(c) time(s) taken for the particle to attain a height of 40 m above the ground. [Take g = 10 ms^{-2}]

2017 JUNE WAEC FURTHER MATHEMATICS PAPER 2 THEORY QUESTIONS

1. (a) If $f(x) = (4 - 5x)/2$ and $g(x) = x + 6$, $x \in R$, find $f \circ g^{-1}$.

(b) P(x, y) divides the line joining (7, −5) and (−2, 7) internally in the ratio 5 : 4. Find the coordinates of P.

2. Evaluate $\int (x - 1)/(x + 1)^2 \, dx$ (limits of integration not clearly legible)

3. (a) Given that $\log_{10} p = a$, $\log_{10} q = b$, $\log_{10} s = c$, express $\log_{10} (p^{10}q^4 / s^5)$ in terms of a, b and c.

(b) The radius of a circle is 6 cm. If the area of the circle is increasing at the rate of 20 $cm^2 s^{-1}$, find, leaving the answer in terms of π, the rate at which the radius is increasing.

4. (a) If (x + 1) and (x − 2) are factors of the polynomial $g(x) = x^4 + ax^3 + bx^2 - 16x - 12$, find the values of a and b.

5. Bottles of the same sizes produced in a factory are packed in boxes. Each box contains 10 bottles. If 8% of the bottles produced are defective, find, correct to two decimal places, the probability that a box chosen at random contains at least three defective bottles.

6. The table shows the heights in cm of some seedlings in a certain garden.

Height (cm)	**36–40**	**41–45**	**46–50**	**51–55**	**56–60**
Frequency	[unclear]	9	21	12	5

(a) Draw a cumulative frequency curve for the distribution. SEE DIAGRAM

(b) Using the curve in 6(a), find the semi-interquartile range.

7. A parallelogram MNQR has vertices M(4, −6), N(10, 2), and Q(8, 16) and R(x, y). Find the coordinates of R.

8. Forces F_1 (18 N, 330°), F_2 (10 N, 090°) and F_3 (25 N, 180°) act on a body at rest. Find, correct to one decimal place, the magnitude and direction of the resultant force.

SECTION B

PART I: PURE MATHEMATICS

9. (a) Simplify $1/(1 - \cos\theta) + 1/(1 + \cos\theta)$ and leave the answer in terms of $\sin\theta$.

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(b) Find the equation of the line joining the stationary points of $y = x^2(x - 3)$ and the distance between them.

10. (a) If $f(x) = (2x - 3)/((x^2 - 1)(x + 2))$,

(i) find the values of x for which f(x) is undefined

(ii) express f(x) in partial fractions

(b) A circle with centre (−3, 1) passes through the point (3, 1). Find the equation.

11. (a) If $f'(x) = \int(4x - x^2)dx$ and f(3) = 21, find f(x).

(b) The second, fourth and eleventh terms of an Arithmetic Progression (A.P) form the first three consecutive terms of a Geometric Progression (G.P). The sum of the third and fifth terms of the A.P. is 20. Find the:

(i) first four terms of the A.P.

(ii) sum of the first ten terms of the A.P.

PART II: STATISTICS AND PROBABILITY

12. (a) In a school, the ratio of those who passed to those who failed in a History test is 4 : 1. If 7 students are selected at random from the school, find, correct to two decimal places, the probability that:

(i) at least 3 students passed the test;

(ii) between 3 and 6 students failed the test.

(b) A fair die is thrown five times, find the probability of obtaining a six three times.

13. The table shows the frequency distribution of the ages of patients in a clinic.

Age (years)	**17–19**	**20–22**	**23–28**	**29–34**	**35–43**
No. of patients	6	9	12	18	18

(a) Draw a histogram for the distribution. SEE DIAGRAM

(b) Find, correct to two decimal places, the mean age of the patients.

PART III: VECTORS AND MECHANICS

14. (a) A body P of mass q kg is suspended by two light inextensible strings AB and DB attached to a horizontal table. The strings are inclined at 30° and 60° respectively to the horizontal and the tension in AB is 48 N. If the system is in equilibrium:

(i) sketch a diagram to represent the information; SEE DIAGRAM

(ii) calculate the tension in DB;

(iii) find the values of q. [Take $g = 10\ ms^{-2}$]

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(b) A ball is thrown vertically upwards with a velocity of 20 ms^{-1}. Find, correct to two decimal places, the: (i) maximum height reached by the ball (ii) time taken to reach the maximum height [Take g = 10 ms^{-2}]

15. (a) Given that m = (6i + 8j) and n = (−8i + 7j), find the:
(i) magnitudes and directions of m and n;
(ii) angle between m and n.

(b) The position vectors of points P, Q, R, S are (−2, 3), (10, 4), (3, 12) and (4, 0) (column vectors) respectively. Show that PQ is perpendicular to RS.

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(ii) angle between m and n.

(b) The position vectors of points P, Q, R, S are (−2, 3), (10, 4), (3, 12) and (4, 0) (column vectors) respectively. Show that PQ is perpendicular to RS.

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2018 WASSCE FURTHER MATHEMATICS THEORY QUESTIONS

SECTION A

1. If

x – 3	**–4**	**3**
5	2	2
2	–4	6 – x

= –24, find the values of x.

2. Given that $\log_3 x - 3 \log_x 3 + 2 = 0$, find the values of x.

3. (a) Using the substitution u = x – 2, write $(x^3 + 5)/(x - 2)^4$ as an expression in terms of u.

(b) Using the answer in 3(a), express $(x^3 + 5)/(x - 2)^4$ in partial fractions.

4. The sum of the first twelve terms of an Arithmetic Progression (A.P) is 168. If the third term is 7, find the values of the common difference and the first term.

5. Two panel of judges, X and Y, rank 8 brand of cooking oil as follow:

Cooking oil type	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
X	8	5	1	7	2	6	3	4
Y	6	3	4	8	5	7	1	2

Calculate the Spearman's rank correlation coefficient.

6. (a) The probability that Kunle solves a particular question is 1/3 while that of Tayo is 1/5. If both of them attempt the question, find the probability that only one of them will solve the question.

(b) A committee of 8 is to be chosen from 10 persons. In how many ways can this be done if there is no restriction?

7. Given that m = 3i – 2j, n = 2i + 3j and p = –i – 6j, find |4m + 2n – 3p|.

8. A body of mass 20 kg moving with a velocity of 80 ms^{-1} collides with another body of mass 30 kg moving with a velocity of 50 ms^{-1}. If the two bodies moved together in the same direction after collision, find their common velocity if they moved in the:

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(a) same direction before collision;

(b) opposite direction before collision.

SECTION B

PURE MATHEMATICS

9. A circle is drawn through the points (3, 2), (−1, −2) and (5, −4). Find the: (a) coordinates of the centre of the circle; (b) radius of the circle; (c) equation of the circle.

10. (a) Solve $2^{(3y+2)} - 7(2^{(2y+2)}) - 31(2^y) - 8 = 0$, $y \in R$.

(b) Find $\int(\sqrt{x} + 1)^2 x \, dx$.

11. (a) (i) Write down the binomial expansion of $(2 - \frac{1}{2}x)^5$ in ascending powers of x.

(ii) Using the expansion in 11(a)(i), find, correct to two decimal places, the value of $(1.99)^5$.

(b) The polynomial $x^3 + qx^2 + rx + 9$, where q and rare constants, has (x + 1) as a factor and has a remainder of −17 when divided by (x + 2). Find the values of q and r.

PART II: STATISTICS AND PROBABILITY

12. Ten coins were tossed together a number of times. The frequency distribution of the number of heads obtained is given in the following table:

Number of Heads	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Frequency	2	7	23	36	11	61	100	12	8	5	3

Calculate, correct to three decimal places, the:

(a) mean number of heads

(b) probability of getting an even number of heads

(c) probability of getting an odd number of heads

13. The probability that Ali, Baba and Katty will gain admission to college are 2/3, 3/4 and 4/5 respectively. Find the probability that:

(a) only Katty and Baba will gain admission

(b) none of them will gain admission

(c) at most two of them will gain admission.

PART III: VECTORS AND MECHANICS

14. The position vectors of points A, B and C with respect to the origin are (8i − 10j), (2i + 6j) and (−10i + 4j) respectively. If ABCN is a parallelogram, find

(a) the position vector of N

(b) |AN| and |AB|

(c) correct to two decimal places, the acute angle between AN and AB.

15. A uniform beam, XY, 4 m long and weighing 350 N rests on two pivots P and Q. It is kept in equilibrium by weights of 80 N attached at X and 1000 N attached at a point between P and Q such that it is 0.6 m from Q. If |XP| = 0.8 m and |PQ| = 2.2 m;

(a) calculate the reactions at P and Q

(b) if the 1000 N weight is replaced with a 1200 N weight, at what point from Q should it be placed in order to maintain the equilibrium.

PART III: VECTORS AND MECHANICS

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(a) the position vector of N

(b) |AN| and |AB|

(c) correct to two decimal places, the acute angle between AN and AB.

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(a) calculate the reactions at P and Q

(b) if the 1000 N weight is replaced with a 1200 N weight, at what point from Q should it be placed in order to maintain the equilibrium.

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2019 WASSCE 2019 FURTHER MATHEMATICS 2 ESSAY

SECTION A

1. Simplify: [625^((3x − 1)/4) + 125^(x − 1)] / 5^(3x − 2)

2. (a) Find the coordinates of the point which divides the line joining (7, −5) and (−2, 7) externally in the ratio 3 : 2.

(b) Without using calculators or mathematical tables, evaluate 2/(1 − √2) − 2/(2 + √2), leaving the answer in the form p + q√n, where p, q and n are integers.

3. How many terms of the series −3 − 1 + 1 + ... add up to 165?

4. If sin X = (p − q)/(p + q), where 0° ≤ X ≤ 90°, find 1 − tan²X.

5. Three forces 10 N, 14 N and 16 N acting on a particle keep it in equilibrium. Find the angle between the forces 10 N and 16 N.

6. A uniform beam, WX, of length 90 cm and weight 50 N is suspended on a pivot, 35 cm from W. It is kept in equilibrium by means of forces T and 20 N applied at Y and Z respectively. |WY| = 10 cm and |XZ| = 10 cm. Find the value of T.

7. The table shows the age distribution in years of a group of people.

Age (in years)	**1–5**	**6–10**	**11–15**	**16–20**	**21–25**	**26–30**
Number of People	18	12	25	15	20	10

Using an assumed mean of 13 years, find the mean age of the people.

8. Two fair dice are thrown together two times. Find the probability of obtaining a sum of seven in the first throw and a sum of four in the second throw.

SECTION B

PART I: PURE MATHEMATICS

9. The curve y = 7 − 6/x and the line y + 2x − 3 = 0 intersect at two points. Find the:

(a) coordinates of the two points;

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(b) equation of the perpendicular bisector of the line joining the two points.

10. (a) Find the range of values of p for which $4x^2 - px + 1 = 0$ has real roots.

(b) (i) Expand $(1 + 3x)^6$ in ascending powers of x.

(ii) using the expansion in 10(b)(i), find, correct to four significant figures, the value of $(1.03)^6$.

11. Given the M: (x, y) → (7y, 3x − y) and N: (x, y) → (2x − y, 5x + 3y),

(a) write down matrices M and N of the linear transformations;

(b) find the image of P(2, −3) under the linear transformation N followed by M;

(c) find the coordinates of the point Q whose image is Q'(2, 4) under the linear transformation N.

PART II: STATISTICS AND PROBABILITY

12. In an examination, 60% of the candidates passed. If 10 candidates are selected at random, find the probability that:

(a) at least two of them failed;

(b) exactly half of them passed;

(c) at most two of them failed.

13. In research, to determine the relationship between performance of students in an entrance examination and subsequent school performance, the results of ten randomly selected students were obtained as follows:

Students	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Performance in Entrance Examination	11	12	8	13	6	15	10	14	17	16
School Performance	5	10	9	7	4	8	6	14	11	12

(a) Calculate the Spearman's rank correlation coefficient.

(b) What would be the researcher's conclusion from the result in 13(a)?

PART III: VECTORS AND MECHANICS

14. A body, moving at 20 ms^{-1} accelerates uniformly at 2½ ms^{-2} for 4 seconds. It continues the journey at this speed for 8 seconds, before coming to rest t seconds after uniform retardation. If the ratio of the acceleration to retardation is 3 : 4,

(a) sketch the velocity-time graph of the journey; SEE DIAGRAM

(b) find t;

(c) find the total distance of the journey.

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15. (a) Given that $m = i - j$, $n = 2i + 3j$ and $2m + n - r = 0$, find $|r|$.

(b) The distance, S metres of a moving particle at any time t seconds is given by $S = 3t - t^3/3 + 9$. Find the: (i) time; (ii) distance travelled; when the particle is momentarily at rest.

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2020: WASSCE FURTHER MATHEMATICS THEORY QUESTIONS

SECTION A

1. A binary operation * is defined on the set of real numbers, R, by p * q = p + q − pq/2, where p, q ∈ R. Find the:
(a) inverse of −1 under *, given that the identity element is zero;
(b) truth set of m * 7 = m * 5.

2. (a) Two functions p and q are defined on the set of real numbers, R, by p: y → 2y + 3 and q: y → y^2 − 2. Find q ∘ p.
(b) How many four-digit odd numbers greater than 4000 can be formed from 1, 7, 3, 8, 2 if repetition is not allowed?

3. Evaluate: $\int_1^2 (3x - 2)^2 \, dx$.

4. If $(3x^2 + 3x - 2)/((x - 1)(x + 1)) = p + Q/(x - 1) + R/(x + 1)$, find the values of Q and R.
(A handwritten rewrite beside it reads: "If $(3x + 1)/((x - 1)(x + 1)) = Q/(x - 1) + R/(x + 1)$ find the values of Q and R.")

5. The table shows the distribution of marks scored by 64 students in a test.

Mark	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	2	2	2	8	13	11	12	10	4

(a) Draw a histogram for the distribution.
(b) Use the histogram to estimate the modal score.

6. (a) A bag contains 10 red and 8 green identical balls. Two balls are drawn at random from the bag, one after the other, without replacement, find the probability that one is red and the other is green.
(b) There are 20% defective bulbs in a large box. If 12 bulbs are selected randomly from the box, calculate the probability that between two and five of them are defective.

7. Forces F_1 (10 N, 090°), F_2 (20 N, 210°) and F_3 (4 N, 330°) act on a particle. Find, correct to one decimal place, the magnitude of the resultant force.

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8. Given that w = 8i + 3j, x = 6i − 5j, y = 2i + 3j and |z| = 41, find z in the direction of w + x − 2y.

SECTION B

Answer four questions only from this section with at least one question from each part.

PART I: PURE MATHEMATICS

9. (a) If (x + 2) is a factor of $g(x) = 2x^3 + 11x^2 - x - 30$, find the zeros of g(x).

(b) Solve $3(2^x) + 3^y = 25$ and $2^x - 3^{(y+1)} = -19$ simultaneously.

10. (a) Find the derivative of $y = x^2(1 + x)^{(1/2)}$ with respect to x.

(b) The centre of a circle lies on the line 2y − x = 3. If the circle passes through P(2, 3) and Q(6, 7), find its equation.

11. (a) Simplify: $\mathbf{(\log_2 8 + \log_2 16 - 4 \log_2 2) / \log_2 16}$

(b) The first, third and seventh terms of an arithmetic progression (A.P) form three consecutive terms of a Geometric Progression (G.P). If the sum of the first two terms of the A.P is 6, find the: (i) first term; (ii) common difference.

PART II: STATISTICS AND PROBABILITY

12. (a) An association is made up of 6 farmers and 8 traders. If an executive body of 4 members is to be formed, find the probability that it will consist of at least two farmers.

(b) The probability of an accident occurring in a given month in factories X, Y and Z are 1/5, 1/12 and 1/6 respectively. Find the probability that the accident will occur in:

(i) none of the factories;

(ii) all the factories;

(iii) at least one factory.

13. The essays of 10 candidates were ranked by three examiners as shown in the table:

Candidate	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Examiner I	1st	3rd	6th	2nd	10th	9th	7th	4th	8th	5th
Examiner II	2nd	1st	3rd	9th	7th	4th	8th	10th	5th	6th

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Examiner III	3rd	2nd	1st	6th	9th	8th	7th	5th	4th	10th

(a) Calculate the Spearman's rank correlation coefficient of the ranks assigned by:

(i) Examiners I and II.

(ii) Examiners I and III.

(iii) Examiners II and III.

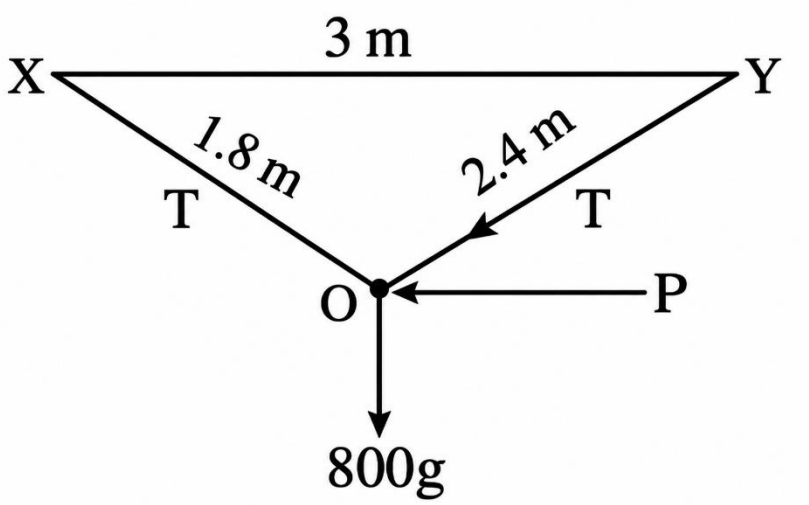
(b) Using the results in 13(a), state which two examiners agree most.

PART III: VECTORS AND MECHANICS

14. (a) A car is moving with a velocity of 10 ms^{-1}. It then accelerates at 0.2 ms^{-2} for 100 m. Find, correct to two decimal places, the time taken by the car to cover the distance.

(b) A particle moves along a straight line such that its distance S metres from a fixed point O is given by $S = t^2 - 5t + 6$, where t is the time in seconds. Find its: (i) initial velocity (ii) distance when it is momentarily at rest.

15. The diagram is that of a light inextensible string of length 4.2 m,



whose ends are attached to two fixed points X and Y, 3 m apart on the same horizontal level. A body of mass 800 g is hung on the string at a point O, 2.4 m from Y. If the system is kept in equilibrium by a horizontal force P acting on the body and the tensions are equal, calculate:

(a) angle XOY

(b) the magnitude of the force P

(c) the tension T in the string.

2021 FURTHER MATHEMATICS

SECTION A [48 marks]

Answer all the questions in this section. All questions carry equal marks.

1. The polynomial $f(x) = 2x^3 + px^2 + qx - 5$ has $(x - 1)$ as a factor and a remainder of 27 when divided by $(x + 2)$, where p and q are constants. Find the values of p and q.

2. Evaluate: $\int_1^9 x(2x - 3)/\sqrt{x}\, dx$.

3. Given that $(p + \frac{1}{2}\sqrt{3})(1 - \sqrt{3})^2 = 3 - \sqrt{3}$, find the value of p.

4. If ${}^{5}yC_2 = 190$, find the value of y.

5. The table shows the distribution of monthly income (in thousands of naira) of workers in a factory.

Monthly Income (₦'000)	**135–139**	**140–149**	**150–154**	**155–164**	**165–169**
Number of workers	20	42	28	38	22

(a) Draw a histogram for the distribution. SEE DIAGRAM

(b) Use your graph to estimate the mode of the distribution.

6. A bag contains 24 mangoes out of which six are bad. If 6 mangoes are selected randomly from the bag with replacement, find the probability that not more than 3 are bad.

7. (a) The speed of a moving bus reduced from 45 ms^{-1} to 5 ms^{-1} with a uniform retardation of 10 ms^{-2}. Calculate the distance covered.

(b) A bucket full of water with mass 16 kg is pulled out of a well with a light inextensible rope. Find its acceleration when the tension in the rope is 240 N. [Take g = 10 ms^{-2}]

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8. Given that x = (−4, 3) and y = (−9, −15) (column vectors), calculate, correct to the nearest degree, the angle between the vectors.

SECTION B [52 marks]

Answer four questions only from this section with at least one question from each part. All questions carry equal marks.

PART I: PURE MATHEMATICS

9. (a) A jogger is training for a 15 km charity race. He starts with a run of 500 metres, then he increases the distance he runs daily by 250 metres.
(i) How many days will it take the jogger to reach a distance of 15 km in training?
(ii) Calculate the total distance he would have run in the training.
(b) The second term of a Geometric Progression (G.P.) is −3. If its sum to infinity is 25/2, find its common ratios.

10. P and Q are two linear transformations in the x−y plane defined by P: (x, y) → (−3x + 6y, 4x + y) and Q: (x, y) → (2x − 3y, −4x − 6y).
(a) Write down the matrices of P and Q.
(b) What is the image of (−2, −3) under the transformation Q?
(c) Obtain a single transformation representing the transformation Q followed by P.
(d) Find the image of (1, 4) when transformed by Q followed by P.
(e) Find the image P′ of the point (−√2, 2√2) under an anticlockwise rotation of 225° about the origin.

11. (a) Find the equation of the normal to the curve $y = (x^2 - x + 1)(x - 2)$ at the point where the curve cuts the x-axis.
(b) The coordinates of the points P, Q and R are (−1, 2), (5, 1) and (3, −4) respectively. Find the equation of the line joining Q and the midpoint of PR.

PART II: STATISTICS AND PROBABILITY

12. A box contains 5 red, 7 blue and 4 green identical bulbs. Two bulbs are picked at random from the box without replacement. Calculate the probability of picking:
(a) same colour of bulbs;
(b) different colour of bulbs;
(c) at least one red bulb.

13. The table shows the frequency distribution of heights (in cm) of pupils in a certain school.

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Heights (cm)	100–109	110–119	120–129	130–139	140–149	150–159	160–169
Frequency	27	58	130	105	50	25	5

(a) (i) Construct a cumulative frequency table.

(ii) Use the table to draw a cumulative frequency curve. SEE DIAGRAM

(b) Using the curve, estimate the: (i) median height; (ii) interquartile range; (iii) percentage of students whose heights are at most 130 cm.

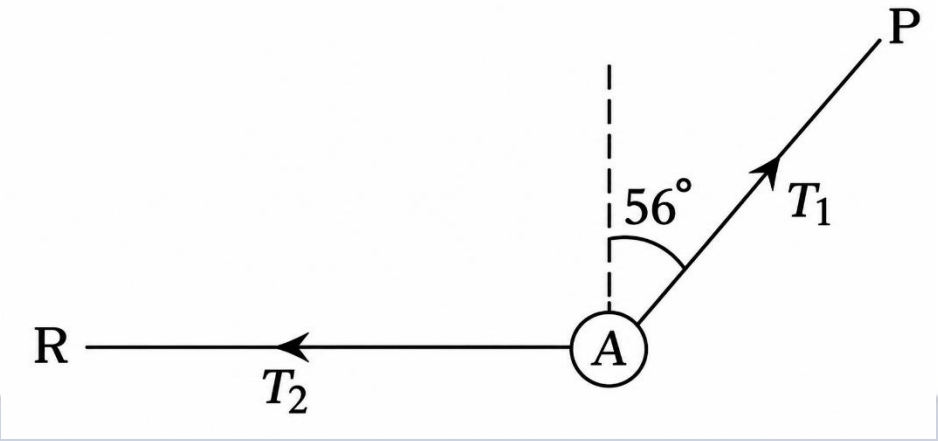
PART III: VECTORS AND MECHANICS

14. The position vectors of P, Q and R with respect to the origin are (4i – 5j), (i + 3j) and (−5i + 2j) respectively. If PQRM is a parallelogram, find:

(a) the coordinates of M;

(b) the acute angle between PM and PQ, correct to the nearest degree.

15. (a) A girl threw a stone horizontally with a velocity of 30 ms^{-1} from the top of a cliff 50 m high. How far from the foot of the cliff does the stone strike the ground? [Take g = 10 ms^{-2}]



(b) A body A, of mass 2 kg is held in equilibrium by means of two strings AP and AR. AP is inclined at 56° to the upward vertical and AR is horizontal. Find the tensions T_1 and T_2 in the strings. [Take g = 10 ms^{-2}]

2022 WASSCE FURTHER MATHEMATICS THEORY QUESTIONS

1. A binary operation * is defined on the set T = {−2, −1, 1, 2} by p * q = $p^2 + 2pq - q^2$, where p, q ∈ T.

(a) Copy and complete the table (only some entries were visible on the page)

*	−2	−1	1	2
−2		7		−8
−1		2	−2	
1	−7			
2		−1		

(b) Using the table in 1(a), find the value of p such that (−2 * p) * 2 = −7.

2. Solve 2^(2x − 1) − 5(2^x) + 2 = 0.

3. Two functions f and g are defined on the set of real numbers, R by f: x → x^2 + 2 and g: x → 1/(x + 2), x ≠ −2. Find the domain of $(g \circ f)^{-1}$.

4. Solve 3cos2x − sin x = 0 for 0° ≤ x ≤ 360°.

5. The probability that Abiola will be late to office on a given day is 2/5. In a given working week of six days, find, correct to three decimal places, the probability that he will:

(a) be late for only 3 days

(b) not be late in the week

(c) be late throughout the six days.

6. The table shows the scores obtained by a group of artistes in a Vocal (X) and instrumental (Y) musical competition.

Vocal (X)	**63**	**69**	**72**	**59**	**82**	**91**	**95**	**68**
Instrumental (Y)	58	61	67	51	53	79	92	57

Calculate the Spearman's rank correlation coefficient between the scores.

7. A body of mass 18 kg is suspended by an inextensible string from a rigid support and is pulled by a horizontal force F until the angle of inclination of the string to the vertical is 35°. If the system is in equilibrium, calculate the:

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(a) value of F

(b) tension in the string (Take g = 10 ms^{-2})

8. Given that p = (8 N, 030°) and q = (9 N, 150°), find, in component form, the unit vector along (p − q).

SECTION B

Answer four questions only from this section with at least one question from each part.

PART I: PURE MATHEMATICS

9. Given that n_4, n_5 and n_6 are the first 3 terms of a linear sequence (A.P.), find the:

(a) values of n

(b) common differences of the sequence.

10. A solid rectangular block has a base which measures 3x cm by 2x cm. The height of the block is y cm and its volume is 72 cm^3.

(a) Express y in terms of x

(b) find:

(i) an expression for the total surface area of the block in terms of x only

(ii) the value of x for which the total surface area has a stationary value.

11. (a) Find the binomial expansion of $(1 + 2x)^7$ and $(1 - 2x)^7$.

(b) Using the result in 11(a), find, correct to three decimal places, the value of $(1.2)^7 - (0.8)^7$.

PART II: STATISTICS AND PROBABILITY

12. A basket contains 12 fruits: orange, apple and avocado pear, all of the same size. The number of oranges, apples and avocado pear forms three consecutive integers. Two fruits are drawn one after the other without replacement. Calculate the probability that

(a) the first is an orange and the second is an avocado pear

(b) both are of the same fruit

(c) at least one is an apple

13. The table shows the corresponding values of two variables X and Y.

X	**14**	**16**	**17**	**18**	**24**	**27**	**28**	**31**	**33**
Y	22	19	15	13	10	12	3	5	3

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(a) plot a scatter diagram to represent the data SEE DIAGRAM

(b) Calculate:

(i) $\bar{x}$, the mean of X and $\bar{y}$, the mean of Y;

(ii) $\bar{x}_1$, the mean of X values below $\bar{x}$ and $\bar{y}_1$, the mean of corresponding Y values below $\bar{x}$.

(c) Draw the line of best fit through $(\bar{x}, \bar{y})$ and $(\bar{x}_1, \bar{y}_1)$. SEE DIAGRAM

(d) From the graph in 13(c), determine the:

(i) relationship between X and Y;

(ii) value of Y when X is 20.

PART III: VECTORS AND MECHANICS

14. (a) A particle initially at rest moves in a straight line with an acceleration of $(10t - 4t^2)$ ms^{-2}. Find the:

(i) velocity of the particle after t seconds;

(ii) average acceleration of the particle during the 4th second.

(b) A load of mass 120 kg is placed on a lift. Calculate the reaction between the floor of the lift and the load when the lift moves upwards:

(i) at a constant velocity

(ii) with an acceleration of 3 ms^{-2} (take g = 10 ms^{-2})

15. The vectors 6i + 8j and 8i − 6j are parallel to OP and OQ respectively. If the magnitude of OP and OQ are 80 units and 120 units respectively, express:

(a) OP and OQ in terms of i and j

(b) |PQ|, in the form $c\sqrt{k}$, where c and k are constants.

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2023 WASSCE FURTHER MATHEMATICS THEORY QUESTIONS

1. If ${}^nC_x = 4[{}^nC_{x+1}]$, find the values of x.

2. The volume of a cube is increasing at the rate of 3½ cm^3s^{-1}. Find the rate of change of the side of the base when its length is 6 cm.

3. The inverse of a function f is given by $f^{-1}(x) = (5x - 6)/(4 - x)$, $x \neq 4$. Find the:

(a) function, f(x);

(b) value of c for which f(c) = 3.

4. The first term of an Arithmetic Progression is −8, the last term is 52 and the sum of terms is 286. Find the:

(a) number of terms in the series;

(b) common difference.

5. The table shows the distribution of the heights (cm) of 60 seedlings in a vegetable garden.

Height (cm)	0.1–0.3	0.4–0.6	0.7–0.9	1.0–1.4	1.5–1.9	2.0–2.2	2.3–2.5
Frequency	6	9	12	15	3	6	9

(a) Draw a histogram for the distribution. SEE DIAGRAM

(b) Use the histogram to estimate the modal height of the seedlings.

6. There are 6 boys and 8 girls in a class. If five students are selected from the class, find the probability that more girls than boys are selected.

7. A bus travels with a velocity of 6 ms^{-1}. It then accelerates uniformly and travels a distance of 70 m. If the final velocity is 20 ms^{-1}, find, correct to one decimal place, the:

(a) acceleration;

(b) time to travel this distance.

8. P is the midpoint of NO and equidistant from MN and MO. If MN = $8i + 3j$ and MO = $14i - 5j$, find MP.

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SECTION B

Answer four questions only from this section, with at least one question from each part.

PART I: PURE MATHEMATICS

9. (a) Find the derivative of $(4x - 7)/x^2$ with respect to x, from the first principle.

(b) Given that tan P = 3/(x − 1) and tan Q = 2/(x + 1), find tan (P − Q).

10. (a) A quadratic polynomial, g(x) has (2x + 1) as a factor. If g(x) is divided by (x − 1) and (x − 2), the remainders are −6 and −5 respectively. Find:

(i) g(x);

(ii) the zeros of g(x).

(b) Find the third term when $(x/2 - 1/x)^n$ (exact expression partly illegible) is expanded in descending powers of x.

11. (a) Express $(8x^2 + 8x + 9)/((x - 1)(2x + 3)^2)$ in partial fractions.

(b) The coordinates of the centre and circumference of a circle are (−2, 5) and 6π units respectively. Find the equation of the circle.

PART II: STATISTICS AND PROBABILITY

12. The table shows the distribution of marks scored by some candidates in an examination.

Marks	**11–20**	**21–30**	**31–40**	**41–50**	**51–60**	**61–70**	**71–80**	**81–90**	**91–100**
Number of candidates	5	39	14	40	57	25	11	8	1

(a) Construct a cumulative frequency table for the distribution.

(b) Draw a cumulative frequency curve for the distribution. SEE DIAGRAM

(c) Use the curve to estimate:

(i) number of candidates who scored marks between 24 and 58

(ii) lowest mark for distinction, if 12% of the candidates passed with distinction.

13. (a) A bag contains 16 identical balls of which 4 are green. A boy picks a ball at random from the bag and replaces it. If this is repeated 5

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times, what is the probability that he:

(i) did not pick a green ball;

(ii) picked a green ball at least three times?

(b) The deviations from a mean of values from a set of data are −2, (m − 1), $(m^2 + 1)$, −1, 2, (2m − 1) and −2. Find the possible values of m.

PART III: VECTORS AND MECHANICS

14. A particle of mass 2 kg moves under the action of a constant force, F N, with an initial velocity $(3i + 2j)\ ms^{-1}$ and a velocity of $(15i - 4j)\ ms^{-1}$ after 4 seconds. Find the:

(a) acceleration of the particle;

(b) magnitude of the force F;

(c) magnitude of the velocity of the particle after 8 seconds, correct to three decimal places.

15. (a) A see-saw pivoted at the middle is kept in balance by weights of Richard, John and Philip such that only Richard whose mass is 60 kg sits on one side. If they sit at distances 2 m, 3 m and 4 m respectively from the pivot and Philip is 15 kg, find the mass of John.

(b) A body of mass 12 kg rests on a rough plane inclined at an angle of 30° to the horizontal. The coefficient of friction between the body and the plane is $(2/3)\sqrt{3}$. A force of magnitude P Newtons acts on the body along the inclined plane. Find the value of P, if the body is at the point of moving:

(i) down the plane;

(ii) up the plane.

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2024: WASSCE FURTHER MATHEMATICS THEORY QUESTIONS

SECTION A

1. If tan x = 1/3, where 180° < x < 270°, evaluate **(sin2x − cosx)/(2tanx + sin2x),** leaving the answer in surd form (radicals).

2. Find the sum of all natural numbers between 403 and 603 which are divisible by 7.

3. Using the trapezium rule with seven ordinates, evaluate, correct to three decimal places, **∫ from 2.4 to 3.6 of $1/\sqrt{(x^2 - 2)}$ dx**.

4. Find the equation of the normal to the curve $y = 7x - 5x^2$ at x = 2.

5. Two events M and N are such that P(M) = 1/2, P(N) = 9/20 and P(M ∩ N) = 11/50, find:
(a) P′(M ∩ N′)
(b) P(M′ ∩ N)

6. The table shows the distribution of distance (in km) of 60 villages from a state capital.

Distance (in km)	0–19	20–29	30–39	40–49	50–69	70–99	100–149
Number of villages	12	7	6	8	5	9	10

Draw a histogram for the distribution. SEE DIAGRAM

7. The magnitudes of two vectors u and v are 10 N and 12 N respectively. If the magnitude of their resultant is 15 N, calculate the angle between them.

8. A body of mass 40 kg is placed on a rough inclined plane which makes an angle of 30° with the horizontal. If a force of 420 N is applied upwards parallel to the plane, find the:
(a) maximum frictional force that will keep the body in equilibrium
(b) coefficient of friction [Take g = 10 ms^{-2}]

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SECTION B

Answer four questions only from this section with at least one question from each part.

PART I: PURE MATHEMATICS

9. A curve is given by $y = 8x + 27/(2x^2)$. Find:

(a) an expression for dy/dx

(b) the coordinates of the stationary point on the curve and the nature of the stationary point;

(c) the equation of the normal to the curve at (2, 2).

10. Three linear transformations, P, Q and R in the oxy plane are defined by

P: $(x, y) \rightarrow (-4x - y, 2x)$

Q: $(x, y) \rightarrow (y, 6x - 9y)$

R: $(x, y) \rightarrow (x - 2y, 3x + 5y)$

(a) Write down the matrices of P, Q and R.

(b) Find:

(i) $2P - 3R + Q$

(ii) QR

(iii) the inverse of the matrix R.

11. (a) Express $\mathbf{9x/((2x + 1)(x^2 + 1))}$ in partial fractions.

(b) If ${}^{2m}P_2 - 10 = {}^{m}P_3$, find the positive value of m.

PART II: STATISTICS AND PROBABILITY

12. The data shows the ordered marks scored by students in a test: 11, 12, (2x + y), (x + 2y), 14 and $(y^2 - 2x)$. Given that the median is 13½, and y is greater than x by 1, find:

(a) the values of x and y

(b) correct to three significant figures, the standard deviation of the distribution.

13. In an examination, 70% of the candidates passed. If 12 candidates are selected at random, find the probability that:

(a) at least two of them failed

(b) exactly half of them passed

(c) not more than one-sixth of them failed.

PART III: VECTORS AND MECHANICS

14. A particle of weight 12 N lying on a horizontal ground is acted upon by forces F_1 = (10 N, 090°), F_2 = (16 N, 180°), F_3 = (7 N, 030°) and F_4 = (12 N, 300°).

(a) Express all the forces acting on the particle as column vectors

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(b) find, correct to two decimal places, the magnitude of the:

(i) resultant force;

(ii) acceleration with which the particle starts to move [take g = 10 ms^{-2}]

15. (a) A boy runs in a line and his displacement at time t seconds after leaving the start point O is X metres, where $20X = 4t^2 + t^3$. Find the:

(i) velocity of the boy when t = 15 seconds

(ii) value of t for which the acceleration of the boy is 8 times his initial acceleration

(b) A body of mass 6 kg moves with a velocity of 7 ms^{-1}. It collides with a second body moving in the opposite direction with a velocity of 5 ms^{-1}. After collision, the two bodies move together with a velocity of 4 ms^{-1}. Find the mass of the second body.

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MARKING SCHEME

(2015 FURTHER MATHEMATICS ESSAY 2 — SECTION A)

Question 1

$a\Delta b = a^3-b^3$

Find $(\sqrt{3}+\sqrt{2})\Delta(\sqrt{3}-\sqrt{2}) = (\sqrt{3}+\sqrt{2})^3-(\sqrt{3}-\sqrt{2})^3$

Recall: $a^3-b^3=(a-b)(a^2+ab+b^2)$

$(\sqrt{3}+\sqrt{2})^3 = (\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})^2 = (\sqrt{3}+\sqrt{2})(5+2\sqrt{6})$

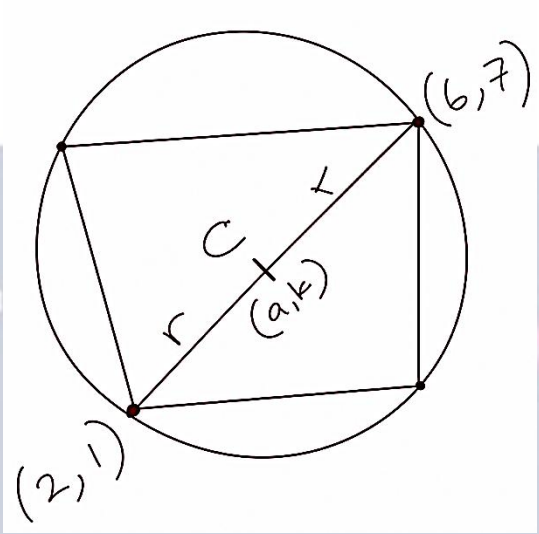
Expanding: $5\sqrt{3}+5\sqrt{2}+2\sqrt{18}+2\sqrt{12} = 5\sqrt{3}+5\sqrt{2}+6\sqrt{2}+4\sqrt{3} = 9\sqrt{3}+11\sqrt{2}$ ...(i)

$(\sqrt{3}-\sqrt{2})^3 = (\sqrt{3}-\sqrt{2})(5-2\sqrt{6})$

Expanding: $5\sqrt{3}-5\sqrt{2}-2\sqrt{18}+2\sqrt{12} = 5\sqrt{3}-5\sqrt{2}-6\sqrt{2}+4\sqrt{3} = 9\sqrt{3}-11\sqrt{2}$ ...(ii)

Combining (i) and (ii): $(9\sqrt{3}+11\sqrt{2})-(9\sqrt{3}-11\sqrt{2}) = 22\sqrt{2}$

Question 2 (Circle Geometry)



Mid-point of the diameter = $\frac{1}{2}(x_1+x_2), \frac{1}{2}(y_1+y_2) = \frac{1}{2}(2+6), \frac{1}{2}(1+7) = (4,4)$

→ Coordinate of Centre = (4,4); a=4, b=4

Radius of the circle = 4

Question 3

$f''(x) = 2$; $f'(x) = \int 2dx = 2x+C$

But $f'(x) = 0$ when $x = -1$

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$0 = 2(-1)+C \rightarrow C = 2$

$\therefore dy/dx = 2x+2$

$f'(x) = 2x-2$

$f(x) = \int(2x-2)dx = x^2-2x+C$

We have $f(x) = -8$ when $x = 0$

$-8 = 0^2-2(0)+C \rightarrow C = -8$

Therefore, $f(x) = x^2-2x-8$

Question 4

$\tan(2x-15)° - 1 = 0$

$\rightarrow \tan(2x-15)° = 1$ (but $1 = \tan 45°$)

$(2x-15)° = 45°$ or $(2x-15)° = 225°$ (3rd quadrant, tan positive there too)

When $(2x-15)°=45°$: $2x=60° \rightarrow x=30°$

When $(2x-15)°=225°$: $2x=240° \rightarrow x=120°$

Question 5

If initial velocity = u, final velocity = 33m/s, acceleration = $3m/s^2$, time = 6s

Using $v = u+at$

$33 = u+3\times6 = u+18 \rightarrow u = 15$ m/s

Distance covered: $S = ut+\frac{1}{2}at^2$

$S = 15\times6+\frac{1}{2}\times3\times6^2 = 90+54$

$\therefore$ Distance travelled = 144m

[SEE DIAGRAM/TABLE — x, y, Kx, Ky, D=Kx-Ky, D^2 columns]

$\Sigma D^2 = 10$

Question 6

Spearman's Rank Correlation Coefficient, r_k, given by:

$r_k = 1 - 6\Sigma d^2/N(N^2-1) = 1 - 6(10)/5(24) = 1-60/120 = 1-0.5 = 0.5$

$\therefore r_k = 0.5$

Question 7

n(Boys)=8, n(Girls)=6, n(E)=14

(a) Pr(selecting a boy 1st) = 8/14

Pr(selecting a boy 2nd) = 7/13

Pr(selecting a girl 1st) = 6/14

Pr(selecting a girl 2nd) = 5/13

Pr(selecting two boys) = $8/14\times7/13 = 28/91$

Pr(selecting two girls) = $6/14\times5/13 = 15/91$

Probability that the two students selected are of same sex = $28/91+15/91 = 43/91$

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(b) Pr(1st student is a boy)=8/14

Pr(2nd student is a girl)=6/13

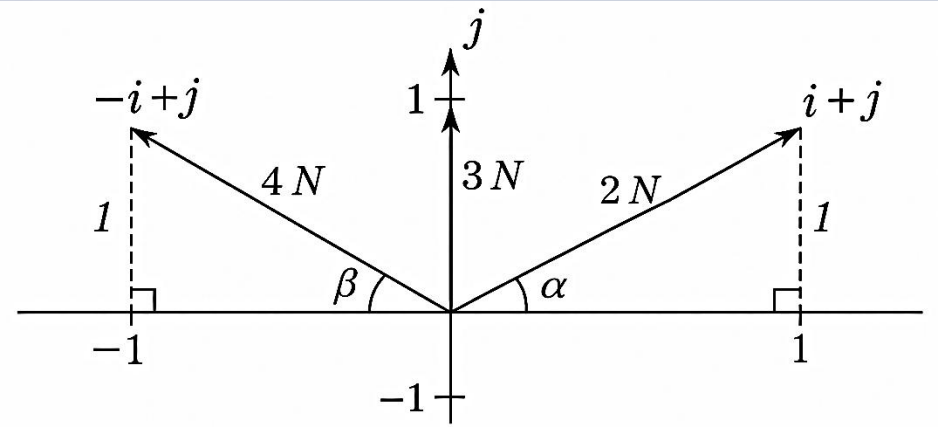
Pr(1st student selected is a girl)=6/14

Pr(2nd student selected is a boy)=8/13

Probability of selecting two students of different sex =

(8/14×6/13)+(6/14×8/13) = 24/91+24/91 = 48/91

Question 8



α = 45°, β = 45°

Unit vector in direction of j = j → force in direction of j = 3j

Unit vector in direction of -1+j = -√2/2 i+√2/2 j

= 4(-√2/2 i+√2/2 j) = -2√2i+2√2j

Unit vector in the 1+j direction = √2/2 i+√2/2 j

→ force in direction of i+j = 2(√2/2 i+√2/2 j) = √2i+√2j

Resultant force: R_1 = 3j-2√2i+2√2j+√2i+√2j = -√2i+(3+3√2)j

|R| = √[(-√2)²+(3+3√2)²] = √[2+9+18√2+18]

= √29+18√2 = √29+18(1.414) = √29+25.452 = √54.452 = 7.37916

∴ |R| = 7.4N (to 1 decimal place)

Question 9

f: x→x²+1; g: x→5-3x

(i) Let y = x²+1 → x² = y-1 → x = √(y-1)

$f^{-1}(y)$ = √(y-1); Hence $f^{-1}(x)$ = √(x-1)

∴ Domain of $f^{-1}(x)$ = {x: x≥1}

(ii) Let y = 5-3x → 3x = 5-y → x = (5-y)/3

$g^{-1}(x)$ = (5-x)/3

Hence, $g^{-1}(2)$ = (5-2)/3 = 3/3 = 1

∴ $g^{-1}(2)$ = 1

(b) $\int_0^2 (x+3)/\sqrt{(x^2+6x+9)}\,dx$

Factorising denominator: $\int_0^2 (x+3)/\sqrt{[(x+3)(x+3)]}\,dx = \int_0^2 (x+3)/(x+3)\,dx = \int_0^2 (1)\,dx$

$= [x]_0^2 = 2-0 = 2$

Question 10

$(\sqrt{5}+4)/(3-2\sqrt{5}) - (2+\sqrt{5})/(4-2\sqrt{5}) = a+b\sqrt{5}$

Rationalising each term:

$= [3\sqrt{5}+2(5)+12+8\sqrt{5}]/[9-20] - [8+4\sqrt{5}+4\sqrt{5}+2(5)]/[16-20]$

$= [11\sqrt{5}+22]/-11 - [8\sqrt{5}+18]/-4$

$= [44\sqrt{5}+88]/-44 - [88\sqrt{5}+198]/-44$

$= [44\sqrt{5}-88\sqrt{5}+88-198]/-44$

$= [-44\sqrt{5}-110]/-44$

$= \sqrt{5}+5/2$, of the form $a+b\sqrt{5}$

$\therefore a = 5/2, b = 1$

b(i) |2 -1 2; 1 3 4; 1 2 1|

$= 2(3-8)+1(1-4)+2(2-3) = -10-3-2 = -15$

(ii) Rewriting: 2x-y+2z=-5; x+3y+4z=1; x+2y+z=-1

x = |-5 -1 2; 1 3 4; -2 2 1| / -15

$= [-5(3-8)+1(1+8)+2(2+6)] / -15$

$= [25+9+16]/-15 = 50/-15 = -3.33$

Unit vector along AB.BC

$\vec{AB}-\vec{BC} = (4,5)-(-3,5) = (7,0)$ or $7i$

$|\vec{AB}\cdot\vec{BC}| = \sqrt{(7^2+0^2)} = \sqrt{49} = 7$

$\{AB/|AB|\cdot BC\} = 1/7(7,0) = (1,0) = i$

(b) $\vec{PQ}=2i+8j$; $\vec{PR}=11i-12j$

$\vec{QR} = \vec{PR}-\vec{PQ} = (11-2)i-12j-8j = 9i-20j$

$\vec{QM} = 3(9i-20j)/(3+7) = (27i-60j)/10 = 2.7i-6j$

$\therefore \vec{PM} = \vec{PQ}+\vec{QM} = (2i+8j)+(2.7i-6j) = 4.7i+2j$

Question 12

(a)

Height/m	f	Class boundaries	Class size
0.4-0.5	2	0.35-0.55	0.2
0.6-0.9	8	0.55-0.95	0.4
1.0-1.2	12	0.95-1.25	0.3
1.3-1.4	6	1.25-1.45	0.2
1.5-1.7	6	1.45-1.75	0.3

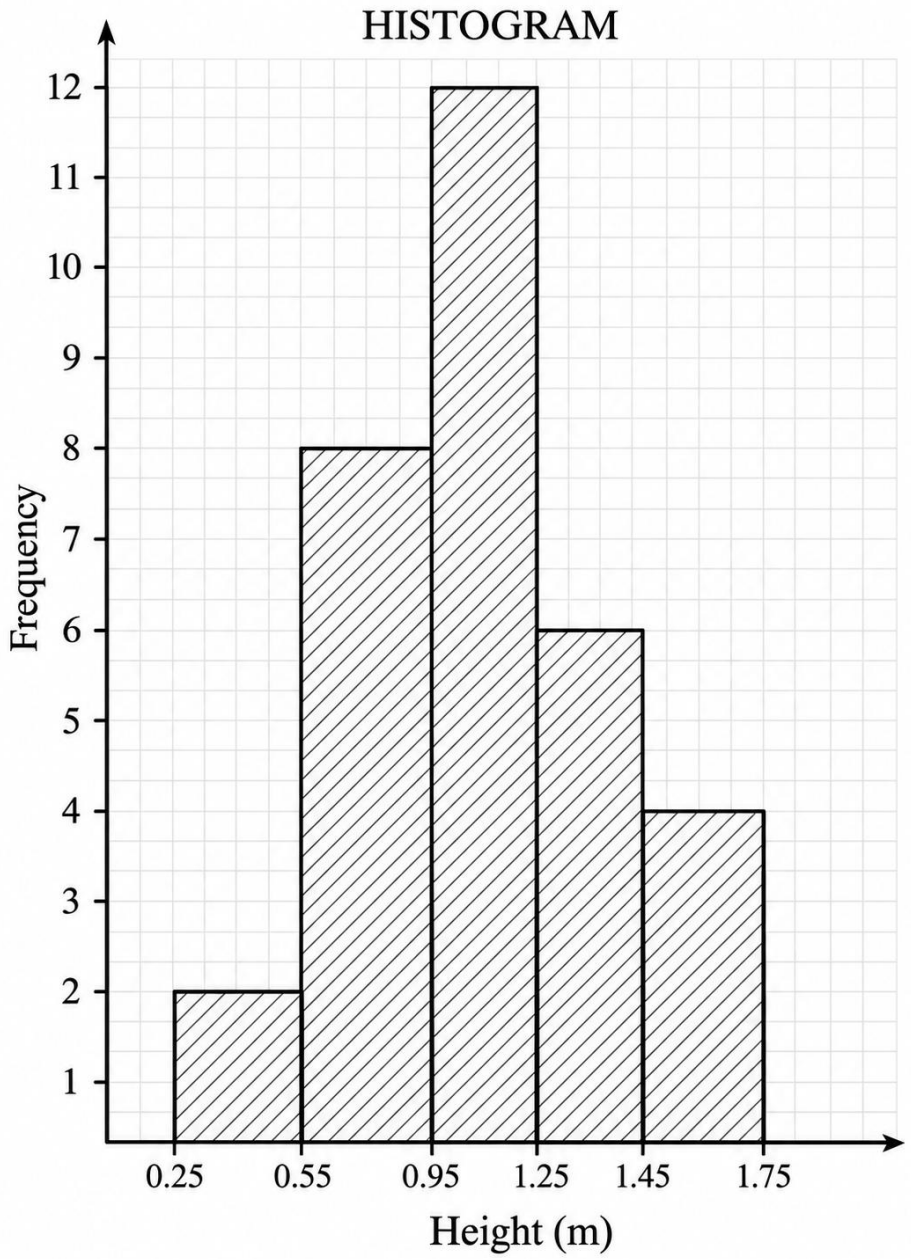
$\Sigma f = 34$

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Class size	f	Area
0.2	2	0.4
0.4	8	1.6
0.3	12	2.4
0.2	6	1.2
0.3	6	1.2

(b) Assumed Mean = 1.10

Height/m	f	Class Mark (x)	d = x-A	fd
0.4-0.5	2	0.45	-0.65	-1.30
0.6-0.9	8	0.75	-0.35	-2.80
1.0-1.2	12	1.10	0.00	0.00
1.3-1.4	6	1.35	0.25	1.50
1.5-1.7	6	1.60	0.50	3.00
Total	**Σf=34**			**Σfd=0.40**



Mean $\bar{x} = A+\Sigma fd/\Sigma f$

$\bar{x} = 1.10+0.40/34 = 1.10+0.0117647 = 1.1117647$

$\therefore \bar{x} \approx 1.1m$ (1 d.p.)

Question 13

(a) For 5 people with Edem and his wife seated together, treat them as one unit → 6 units

$n!\times r! = 6!\times 2! = 720\times 2 = 1440$ ways

(b) Red=4, Black=5, Total=9

Pr(Red)=4/9, Pr(Black)=5/9

Binomial $(R+B)^5$, coefficients 1,5,10,10,5,1

$(R+B)^5 = R^5+5R^4B+10R^3B^2+10R^2B^3+5RB^4+B^5$

(i) $Pr(3Red,2Black) = 10R^3B^2 = 10(4/9)^3(5/9)^2$

$= 10\times 64/729 \times 25/81 = 16,000/59,049$

$Pr(\text{at most } 2B) = R^5+5R^4B+10R^3B^2$

$= 1024/59049+6400/59049+16000/59049 = 23,424/59,049 = 0.3966875$

$(1+x)^4$: coefficients 1 4 6 4 1 → $= 1+4x+6x^2+4x^3+x^4$

(ii) $(5/4)^4 = (1.25)^4 = (1+0.25)^4$, where x=0.25

$= 1+4(0.25)+6(0.25)^2+4(0.25)^3+(0.25)^4$

$= 1+1.00+0.375+0.0625+0.003906 = 2.441406 \approx 2.441$ (3 d.p.)

(b) $T_1=a=7$; $T_2=a+d=ar$; $T_5=a+4d=ar^2$

Terms: 7, 7+d, 7+4d (G.P form)

Common ratio $r = (7+d)/7 = (7+4d)/(7+d)$

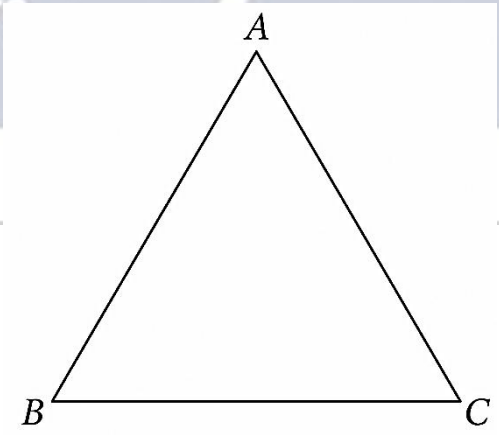
Cross-multiplying: $(7+d)(7+d) = 7(7+4d)$

$49+14d+d^2 = 49+28d \rightarrow d^2-14d = 0$

$d(d-14)=0 \rightarrow d=0$ or $d=14$

Hence, common difference = 14

Question 14



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(a) $\vec{AB}=(4,5)$; $\vec{BC}=(-3,5)$

In component form: $\vec{AB}=4i+5j$; $\vec{BC}=-3i+5j$

$\vec{AC} = \vec{AB}+\vec{BC} = (4i+5j)+(-3i+5j) = i+10j$

(b)(i) Direction Cosine (Dot Product)

$\cos\theta = a\cdot b/(|a||b|)$

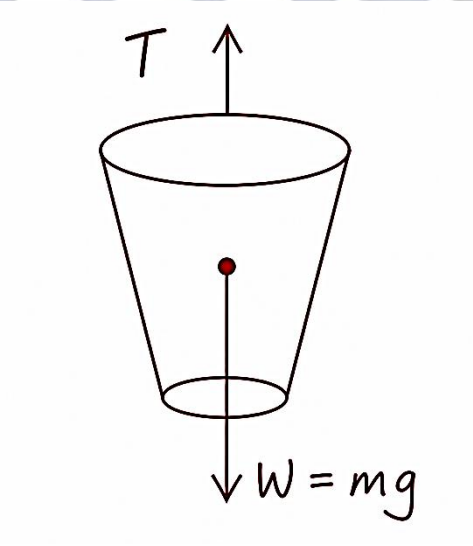
$\vec{AD} = (4i+5j)\cdot(i+10j) / [(\sqrt{4^2+5^2})(\sqrt{1^2+10^2})]$

$\cos\theta = [4(1)+5(10)] / [(\sqrt{41})(\sqrt{101})] = 54/\sqrt{4141} = 54/64.3506 = 0.83915$

$\theta = \cos^{-1}(0.83915) = 32.9495° \approx 32.95°$

∴ Angle between AB and AC = 32.95°

Question 15



(a) Let T = Tension in rope, W = weight of bucket = mg

T = W+ma → T-W = ma

150-8×10 = 8×a → 150-80 = 8a → 70 = 8a

$a = 70/8 = 8.75\ m/s^2$

∴ Acceleration when T=150N is $8.75\ m/s^2$

(b)(i) Recall F=ma; but $v^2=u^2+2as$

$25^2 = 15^2+2\times a\times 50$ → 625 = 225+100a → $a = 400/100 = 4\ m/s^2$

Hence, F=ma = 12×4 → F = 48N

(ii) $v^2=u^2+2as$ (at speed of 35m/s)

$35^2 = 15^2+2\times 4\times s$ → 1225 = 225+8s → 8s = 1000 → s = 125

Hence, Distance covered = 125m

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(2016 FURTHER MATHEMATICS ESSAY 2 — SECTION A)

QUESTION 1

$m * n = m + n + 2$

(a) Let e be the identity element under *.

$m * e = e * m = m$

$m + e + 2 = m$

$e = m - m - 2$

$e = -2$

(b) Let n^{-1} be the inverse of n under *.

$n * n^{-1} = e$

$n + n^{-1} + 2 = -2$

$n^{-1} = -2 - 2 - n = -4 - n$

$n^{-1} = -n - 4$

QUESTION 2

For (5, 2), (−4, k) and (2, 1) to lie on a straight line, the gradient must be the same between any two of the points.

Points (5, 2) and (2, 1):

$M_1 = (1 - 2)/(2 - 5) = -1/-3 = 1/3$

Points (5, 2) and (−4, k):

$M_2 = (k - 2)/(-4 - 5) = (k - 2)/-9$

Since $M_1 = M_2$:

$1/3 = (k - 2)/-9$

$1(-9) = 3(k - 2)$

$-9 = 3k - 6$

$-9 + 6 = 3k$

$-3 = 3k$

$k = -1$

QUESTION 3

(a) $f(x + 2) = 6x^2 + 5x - 8$

To find f(5), set $x + 2 = 5$, so $x = 3$.

$f(3) = 6(3^2) + 5(3) - 8$

$= 6(9) + 15 - 8$

$= 54 + 15 - 8$

$= 69 - 8$

$= 61$

(b) Simplify $(7\sqrt{2} + 3\sqrt{3})/(4\sqrt{2} - 2\sqrt{3})$ by rationalising the denominator.

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= (7√2 + 3√3)/(4√2 − 2√3) × (4√2 + 2√3)/(4√2 + 2√3)

= [28(2) + 14√6 + 12√6 + 6(3)] / [16(2) + 8√6 − 8√6 − 4(3)]

= (56 + 26√6 + 18)/(32 − 12)

= (74 + 26√6)/20

= 74/20 + (26/20)√6

= 37/10 + (13/10)√6

QUESTION 4

$f(x) = (2x - 5)(x^2 + 5x + 1) + 30x + 16$

Expanding:

$2x^3 + 10x^2 + 2x - 5x^2 - 25x - 5 + 30x + 16$

Collecting like terms:

$= 2x^3 + 10x^2 - 5x^2 + 2x - 25x + 30x + 16 - 5$

$= 2x^3 + 5x^2 + 7x + 11$

Comparing this with $2x^3 + mx^2 + nx + 11$ gives m = 5 and n = 7.

QUESTION 5

Pr(Ago gains admission) = 4/5, so Pr(Ago will not gain admission) = 1 − 4/5 = 1/5

Pr(Sulley gains admission) = 3/4, so Pr(Sulley will not gain admission) = 1 − 3/4 = 1/4

Pr(Musa gains admission) = 2/3, so Pr(Musa will not gain admission) = 1 − 2/3 = 1/3

(a) Pr(none of them gains admission) = 1/5 × 1/4 × 1/3 = 1/60

(b) Pr(only Ago and Sulley will gain admission) = 4/5 × 3/4 × 1/3 = 12/60 = 1/5

QUESTION 6

(a) Mean, $\bar{x} = \Sigma x/n$

(1 + 4 + k + k + 4 + 11)/5 = k + 1

Collecting like terms:

(2k + 20)/5 = k + 1

Cross-multiplying:

5(k + 1) = 2k + 20

5k + 5 = 2k + 20

5k − 2k = 20 − 5

3k = 15

k = 15/3 = 5

(b) Standard deviation: substituting k = 5 to generate the table.

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Mean $\bar{x} = k + 1 = 5 + 1 = 6$

x	$d = x - \bar{x}$	d^2
1	$1 - 6 = -5$	25
4	$4 - 6 = -2$	4
5	$5 - 6 = -1$	1
9	$9 - 6 = 3$	9
11	$11 - 6 = 5$	25
		$\Sigma d^2 = 64$

S.D. = $\sqrt{(\Sigma d^2/n)} = \sqrt{(64/5)}$

$= \sqrt{12.8}$

$= 3.5777088$

S.D. = 3.5778

QUESTION 7

(a) $m_1 = 3$ kg, $u_1 = 8$ m/s; $m_2 = ?$, $u_2 = 5$ m/s; common velocity Vc = 6 m/s

Momentum before collision = $m_1u_1 + m_2u_2 = 3 \times 8 + m_2 \times 5$

Momentum after impact = $(m_1 + m_2)Vc$

Applying the principle of conservation of linear momentum:

$3 \times 8 + m_2 \times 5 = (m_1 + m_2)Vc$

$24 + 5m_2 = (3 + m_2)6$

$24 + 5m_2 = 18 + 6m_2$

Collecting like terms:

$6m_2 - 5m_2 = 24 - 18$

$m_2 = 6$ kg

(b) The 2nd body (6 kg) is moving in the opposite direction with a velocity of 5 m/s, so $u_2 = -5$ m/s.

Applying the conservation principle of linear momentum:

$3 \times 8 + 6(-5) = (m_1 + m_2)V$

$24 - 30 = (3 + 6)V$

$9V = -6$

$V = -6/9 = -2/3 = -0.67$ m/s

The common velocity = 0.67 m/s in the direction of the 6 kg body.

QUESTION 8

p = (5, 3), q = (−1, 2), r = (17, 5)

(a) $(17, 5) = \alpha(5, 3) + \beta(-1, 2)$

Solving by expansion:

$5\alpha - \beta = 17$ .....(1) × 2

$3\alpha + 2\beta = 5$ .....(2) × 1

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10α − 2β = 34

3α + 2β = 5

Adding both equations: 13α = 39

α = 39/13 = 3

Put α = 3 in equation (1):

5(3) − β = 17

15 − 17 = β

β = −2

(b) Substituting in the original equation r = αp + βq:

βq = r − αp

q = (r − αp)/β = (r − 3p)/−2

q = −½r + (3/2)p

SECTION B

QUESTION 9

(a) Evaluate (log 27 − 3 log 5√5)/log 0.6

The page reads as follows:

= (3/2 log 3^3 − 3 log 5^(3/2)) / log (6/10)

= ($\log_3$ 3^(3×3/2) − log $(5^{(3/2)})^3$) / log (3/5)

= (log 3^(9/2) − log 5^(9/2)) / log (3/5)

= (9/2 log 3 − 9/2 log 5) / (log 3 − log 5)

Factorising:

= (9/2)(log 3 − log 5) / (log 3 − log 5)

= 9/2

(The first two lines are hard to read in the photo, so check them against your copy.)

(b)(i) (x, y) → (x + 2y, −x + y), which in matrix form is:

(x, y) → [1 2; −1 1](x, y)

This gives the matrix A = [1 2; −1 1].

(x, y) → (2x + 3y, x + 2y), which in matrix form is:

(x, y) → [2 3; 1 2](x, y)

This is equivalent to B = [2 3; 1 2].

The two matrices of transformation are A = [1 2; −1 1]; B = [2 3; 1 2].

(b)(ii) Point P(−2, 2) = (−2, 2)

Under A:

[1 2; −1 1](−2, 2) = (−2×1 + 2×2, −2×1 + 2×1) = (−2 + 4, 2 + 2) = (2, 4)

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Under B:

[2 3; 1 2](2, 4) = (2×2 + 4×3, 2×1 + 4×2) = (4 + 12, 2 + 8) = (16, 10)

Therefore, the image of P(−2, 2) under the linear transformation A followed by B = (16, 10).

QUESTION 10

(a)(i) Expansion of $(1 + x)^7$

Coefficients: 1, 7, 21, 35, 35, 21, 7, 1

$= 1(1^7)(x^0) + 7(1^6)(x^1) + 21(1^5)(x^2) + 35(1^4)(x^3) + 35(1^3)(x^4) + 21(1^2)(x^5) + 7(1^1)(x^6) + 1(1^0)(x^7)$

$= 1 + 7x + 21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7$

(a)(ii) The 5th, 6th and 7th terms are $35x^4$, $21x^5$, $7x^6$

i.e. 35, 21, 7

Common difference d = 21 − 35 = 7 − 21 = −14

Hence d = −14

(b) $\int_1^5 \sqrt{(2x + 8x^{-2})}\, dx$, using the trapezium rule.

h = (upper limit − lower limit)/ordinates = (5 − 1)/4 = 1

Let $y = \sqrt{(2x + 8x^{-2})}$

x	**1**	**2**	**3**	**4**	**5**
2x	2	4	6	8	10
$8x^{-2}$	8	2	0.888	0.500	0.320
√y	3.162	2.449	2.625	2.915	3.212

= ½(3.162 + 2.449) + ½(2.449 + 2.625) + ½(2.625 + 2.915) + ½(2.915 + 3.212)

= ½(5.611) + ½(5.074) + ½(5.54) + ½(6.127)

= 2.8055 + 2.537 + 2.77 + 3.0635

= 11.176

$\int_1^5 \sqrt{(2x + 8x^{-2})}\, dx$ = 11.18 (2 decimal places)

QUESTION 11

(a) $^kP_2 = 72$

$k!/(k - 2)! = 72$

$k(k - 1)(k - 2)!/(k - 2)! = 72$

$k(k - 1) = 72$

$k^2 - k - 72 = 0$

Factorising:

$k^2 - 9k + 8k - 72 = 0$

$k(k - 9) + 8(k - 9) = 0$

$(k + 8)(k - 9) = 0$

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Either $k + 8 = 0$, so $k = -8$, or $k - 9 = 0$, so $k = 9$.

Since k cannot be negative, $k = 9$.

(b) Solve $2\cos^2\theta - 5\cos\theta = 3$

$2\cos^2\theta - 5\cos\theta - 3 = 0$

Let $\cos\theta$ be p:

$2p^2 - 5p - 3 = 0$

Factorising:

$(2p + 1)(p - 3) = 0$

Either $2p + 1 = 0$, so $p = -½$, or $p - 3 = 0$, so $p = 3$.

Hence $p = -½$, because $\cos\theta$ cannot be greater than 1 (the page says "3").

If $\cos\theta = -½$:

$\theta = \cos^{-1}(-½) = 120°$ or $240°$

Marks range	Marks up to	Frequency	Cumulative frequency
1 – 10	10.5	3	3
11 – 20	20.5	17	20
21 – 30	30.5	41	61
31 – 40	40.5	85	146
41 – 50	50.5	97	243
51 – 60	60.5	115	358
61 – 70	70.5	101	459
71 – 80	80.5	64	523
81 – 90	90.5	21	544
91 – 100	100.5	6	550

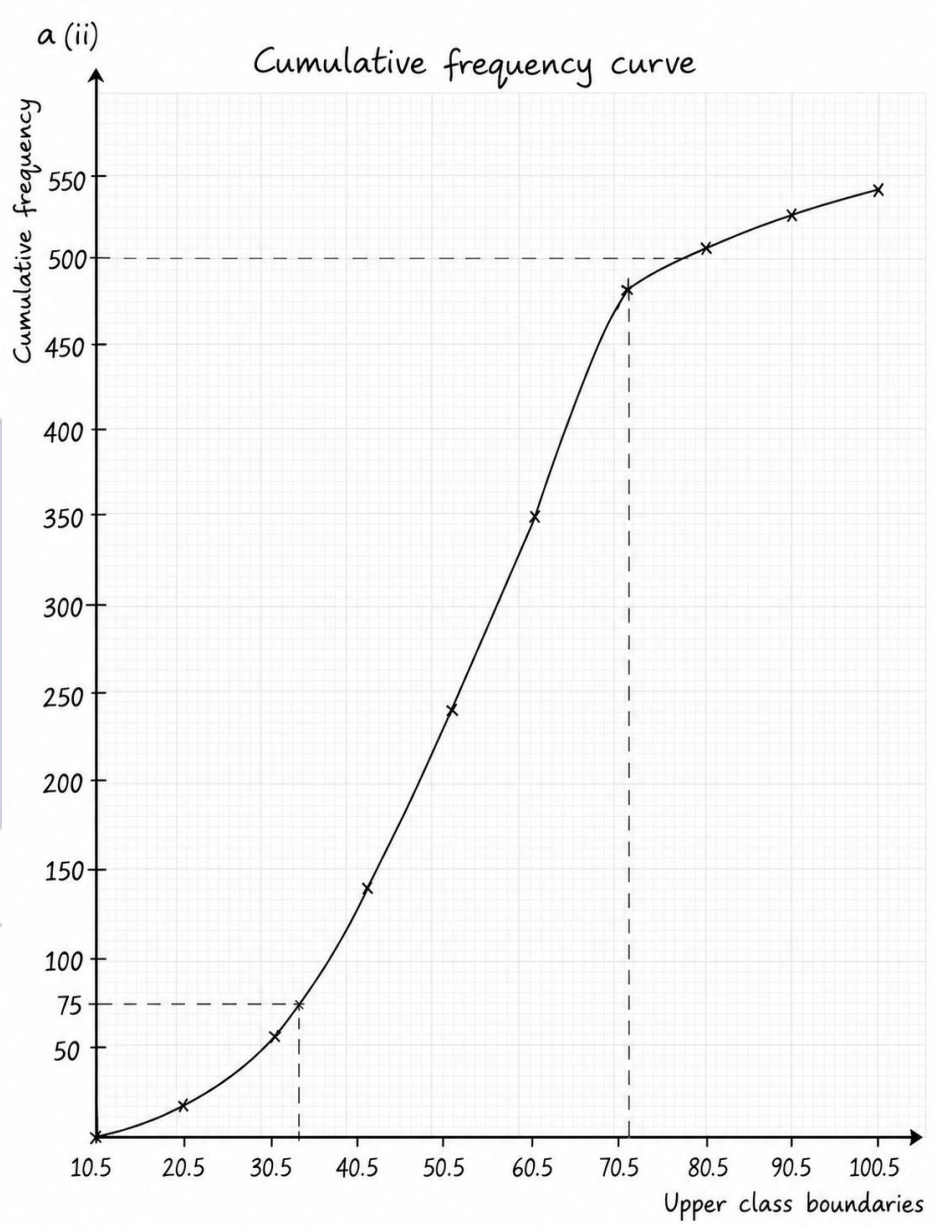
(a)(ii) Cumulative frequency curve (x-axis: upper class boundary; y-axis: cumulative frequency)

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PART II

QUESTION 12

(a)(i) Frequency table



(b)(i) Number of students who scored marks from 32 to 74, from the graph:

= 505 − 75

= 430 students

(b)(ii) If 18% of the students failed:

(18/100 × 550)th = 99th mark

From the graph, the 99th mark = 35.5

(b)(iii) If 8% of the students passed with distinction:

8/100 × 550 = 44 students

550 − 44 = 506

From the graph, the lowest mark for distinction = 77.5

QUESTION 13

(a)(i) Number of ways of arranging 2 Mathematics books, 5 Physics books and 3 Chemistry books, since the three different kinds of books are kept in order:

= 3! 2! 5! 3!

= (3 × 2 × 1)(2 × 1)(5 × 4 × 3 × 2 × 1)(3 × 2 × 1)

= 6 × 2 × 120 × 6

= 8640

(a)(ii) If only the Physics books must stand together:

6! × 5! ways

= (6 × 5 × 4 × 3 × 2 × 1) × (5 × 4 × 3 × 2 × 1)

= 720 × 120

= 86,400 arrangements

(b) 13 out of 20 speak English.

Let p denote those who speak English and q denote those who do not speak English.

p = 13/20, q = 1 − 13/20 = 7/20

The binomial distribution is $(p + q)^8$.

Coefficients: 1, 8, 28, 56, 70, 56, 28, 8, 1

$= p^8 + 8p^7q + 28p^6q^2 + 56p^5q^3 + 70p^4q^4 + 56p^3q^5 + 28p^2q^6 + 8pq^7 + q^8$

P(at least 3 speak English) = P(≥ 3)

$= 1 - \{28p^2q^6 + 8pq^7 + q^8\}$

$= 1 - \{28(13/20)^2(7/20)^6 + 8(13/20)(7/20)^7 + (7/20)^8\}$

= 1 − {(28 × 169 × 117,649)/(400 × 64,000,000) + (8 × 13 × 823,543)/(20 × 1,280,000,000) + 5,764,801/25,600,000,000}

= 1 − (556,715,068 + 85,648,472 + 5,764,801)/25,600,000,000

= 1 − 648,128,341/25,600,000,000

= 1 − 0.0253175

= 0.97468

= 0.975 (3 significant figures)

PART III

QUESTION 14

(a) r = αi + βj, s = 2i − j, m = 3i + 2j, n = i + j

|r| = 3|s|

$\sqrt{(\alpha^2 + \beta^2)} = 3\sqrt{(2^2 + (-1)^2)}$

$\sqrt{(\alpha^2 + \beta^2)} = 3\sqrt{(4 + 1)}$

Squaring both sides:

$\alpha^2 + \beta^2 = 3^2(5)$

$\alpha^2 + \beta^2 = 45$ .....(1)

Again, m − n = 2i + j, so

β/α = ½ .....(2)

Comparing equations (1) and (2): 2β = α, such that

$(2\beta)^2 + \beta^2 = 45$

$4\beta^2 + \beta^2 = 45$

$5\beta^2 = 45$

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$\beta^2 = 45/5 = 9$

$\beta = \sqrt{9} = \pm 3$

Put $\beta = 3$ in equation (2):

$3/\alpha = ½$

$\alpha = 3 \times 2 = 6$

Hence $\alpha = 6$, $\beta = 3$

(b) $(r - s) = (6i + 3j) - (2i - j)$

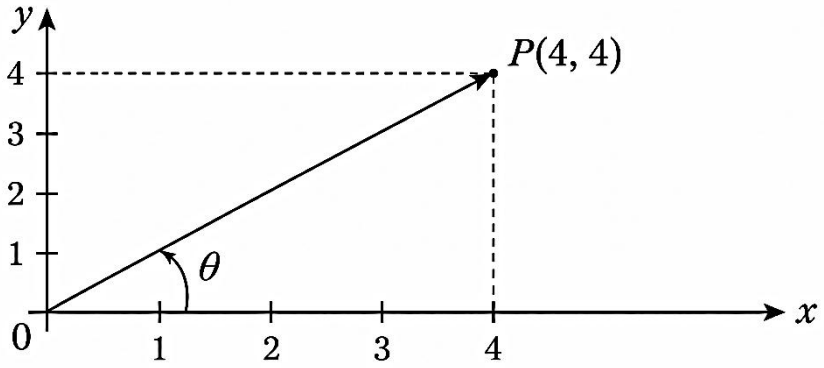
$= 6i + 3j - 2i + j$

$= 4i + 4j$

$|r - s| = \sqrt{(4^2 + 4^2)} = \sqrt{(16 + 16)} = \sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

Direction of $4i + 4j$ in the x-y plane:

(P(4, 4) plotted, with angle θ at the origin)



From the diagram:

$\tan\theta = 4/4 = 1$

$\theta = \tan^{-1}(1) = 45°$

Hence the magnitude of $(r - s) = 4\sqrt{2}$ units in the direction of 045° on the positive x-axis.

QUESTION 15

(a) $u = 30$ m/s, $g = -10$ m/s², $v = 0$

Using $v^2 = u^2 + 2gs$:

$0^2 = 30^2 + 2(-10)s$

$0 = 900 - 20s$

$20s = 900$

$s = 900/20 = 45$ m

Maximum height reached = 45 m

(b) $v = u + gt$

$0 = 30 + (-10)t$

$10t = 30$

$t = 30/10 = 3$ seconds

Hence time of flight = 2(3) = 6 secs

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(c) $s = ut + \frac{1}{2}gt^2$

$40 = 30t + \frac{1}{2}(-10)t^2$

$40 = 30t - 5t^2$

Dividing through by 5:

$t^2 - 6t + 8 = 0$

Solving by factorisation:

$(t - 2)(t - 4) = 0$

Either $t - 2 = 0$ or $t - 4 = 0$

$t = 2$ seconds or $t = 4$ seconds

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2017 — WASSCE JUNE FURTHER MATHEMATICS 2 (ESSAY)
ANSWERS / SOLUTIONS

SECTION A

1(a)

$f(x) = (4 - 5x)/2$; $g(x) = x + 6$.

Let $y = x + 6$.

$y - 6 = x$.

Therefore, $g^{-1}(x) = x - 6$.

Hence:

$(f \circ g^{-1})(x) = f[g^{-1}(x)]$

$= [4 - 5(x - 6)]/2$

$= (4 - 5x + 30)/2$

$= (-5x + 34)/2$

$= -5x/2 + 17$.

Answer: $-5x/2 + 17$.

1(b)

Let (x_p, y_p) denote the coordinates of P.

$x_p = (m_1x_2 + m_2x_1)/(m_1 + m_2)$

$= [5(-2) + 4(7)]/(5 + 4)$

$= (-10 + 28)/9$

$= 18/9$

$= 2$.

Similarly:

$y_p = (m_1y_2 + m_2y_1)/(m_1 + m_2)$

$= [5(7) + 4(-5)]/(5 + 4)$

$= (35 - 20)/9$

$= 15/9$

$= 5/3$.

Coordinates of P = (2, 5/3).

2

Evaluate:

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$\int_1^3 [(x - 1)/(x + 1)^2]\, dx$.

Let $u = x + 1$.

Then $x = u - 1$ and $du/dx = 1$.

Therefore, $dx = du$.

When $x = 1$, $u = 2$.

When $x = 3$, $u = 4$.

Thus:

$\int_2^4 [(u - 1 - 1)/u^2]\, du$

$= \int_2^4 [(u - 2)/u^2]\, du$

$= \int_2^4 (1/u - 2u^{-2})\, du$

$= [\ln u + 2/u]_2^4$

$= (\ln 4 + 2/4) - (\ln 2 + 2/2)$

$= (1.386 + 0.5) - (0.693 + 1)$

$= 1.886 - 1.693$

$= 0.193$.

Answer: 0.193.

3(a)

$\log_{10}[(p^{(1/3)}q^4)/s^2]$

$= (1/3)\log_{10}p + 4\log_{10}q - 2\log_{10}s$.

Recall:

$\log_{10}p = a \Rightarrow p = 10^a$.

$\log_{10}q = b \Rightarrow q = 10^b$.

$\log_{10}s = c \Rightarrow s = 10^c$.

Substituting:

Answer: $a/3 + 4b - 2c$.

3(b)

Area, $A = \pi r^2$.

Therefore:

$dA/dr = 2\pi r$.

By the chain rule:

$dA/dt = (dA/dr)(dr/dt)$

$= 2\pi r(dr/dt)$.

Substituting $dA/dt = 20$ and $r = 6$:

$20 = 2\pi \times 6 \times dr/dt$.

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$20 = 12\pi(dr/dt)$.

$dr/dt = 20/(12\pi)$

$= 5/(3\pi)$ cm/s.

Answer: $5/(3\pi)$ cm/s.

4

$g(x) = x^4 + ax^3 + bx^2 - 16x - 12$.

Since $x + 1$ is a factor:

$g(-1) = 0$.

$(-1)^4 + a(-1)^3 + b(-1)^2 - 16(-1) - 12 = 0$.

$1 - a + b + 16 - 12 = 0$.

$-a + b + 5 = 0$.

Therefore:

$-a + b = -5$. … (1)

Since $x - 2$ is a factor:

$g(2) = 0$.

$2^4 + a(2^3) + b(2^2) - 16(2) - 12 = 0$.

$16 + 8a + 4b - 32 - 12 = 0$.

$8a + 4b - 28 = 0$.

Dividing by 4:

$2a + b - 7 = 0$.

Therefore:

$2a + b = 7$. … (2)

Subtracting equation (2) from equation (1):

$-3a = -12$.

$a = 4$.

Substituting into equation (2):

$2(4) + b = 7$.

$8 + b = 7$.

$b = 7 - 8 = -1$.

Answer: $a = 4$ and $b = -1$.

---

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5

Let the probability of a defective bottle be p.

p = 8/100 = 0.08.

Let the probability of a non-defective bottle be q.

q = 1 − 0.08

= 0.92

= 92/100.

For a box of 10 bottles, the binomial expansion is $(p + q)^{10}$.

The coefficients are:

1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1.

Thus:

$(p + q)^{10}$

$= p^{10} + 10p^9q + 45p^8q^2 + 120p^7q^3 + 210p^6q^4$

$252p^5q^5 + 210p^4q^6 + 120p^3q^7 + 45p^2q^8 + 10pq^9 + q^{10}$.

Probability of at least three defective bottles:

P(X ≥ 3)

$= 1 - (45p^2q^8 + 10pq^9 + q^{10})$

$= 1 - [45(8/100)^2(92/100)^8 + 10(8/100)(92/100)^9 + (92/100)^{10}]$

= 1 − (0.147807 + 0.377729 + 0.434389)

= 1 − 0.959925

= 0.040075

≈ 0.04, to two decimal places.

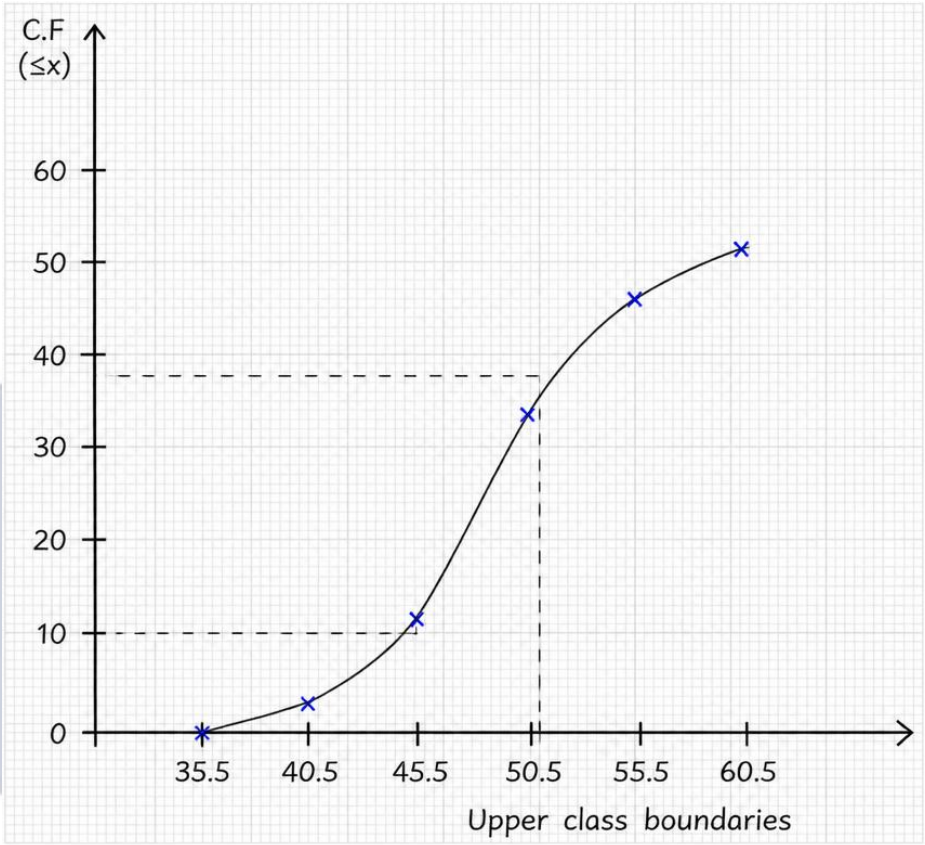
Answer: 0.04.

6(a)

Height (cm)	Frequency	Upper class boundary	Cumulative frequency
36–40	3	40.5	3
41–45	9	45.5	12
46–50	21	50.5	33
51–55	12	55.5	45
56–60	5	60.5	50

Cumulative frequency curve:

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From the curve:

Q_1 = 45.5 cm.

Upper quartile:

Q_3 = the (3n/4)th height

= (3 × 50/4)th height

= 37.5th height.

From the curve:

Q_3 = 53.5 cm.

Semi-interquartile range:

$(Q_3 - Q_1)/2$

= (53.5 − 45.5)/2

= 8/2

= 4.0 cm.

Answer: 4.0 cm.

6(b)Lower quartile:

Q_1 = the (n/4)th height

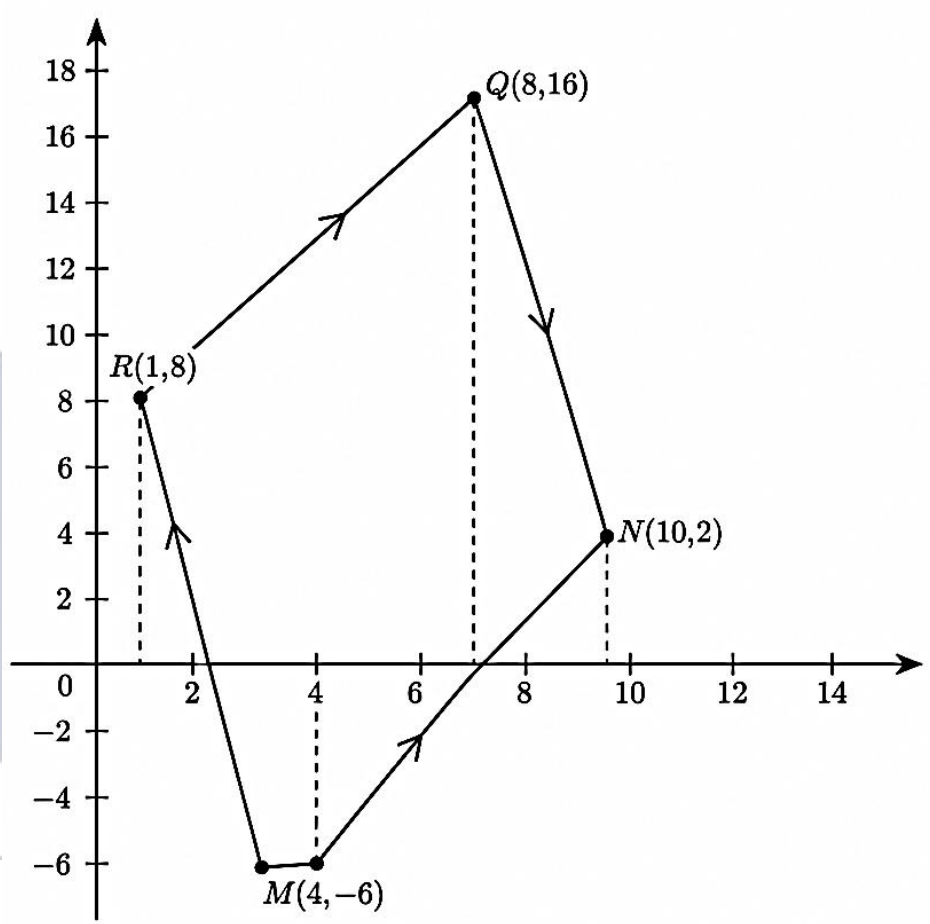
= (50/4)th height

= 12.5th height.

7

The coordinates are:

M(4, −6), N(10, 2), Q(8, 16), R(x, y).



Vector MN = N − M

= (10, 2) − (4, −6)

= (6, 8).

Similarly:

Vector RQ = Q − R

= (8, 16) − (x, y)

= (8 − x, 16 − y).

Since MN and RQ are equal opposite sides of a parallelogram:

(8 − x, 16 − y) = (6, 8).

Therefore:

8 − x = 6.

x = 8 − 6 = 2.

Also:

16 − y = 8.

y = 16 − 8 = 8.

Coordinates of R = (2, 8).

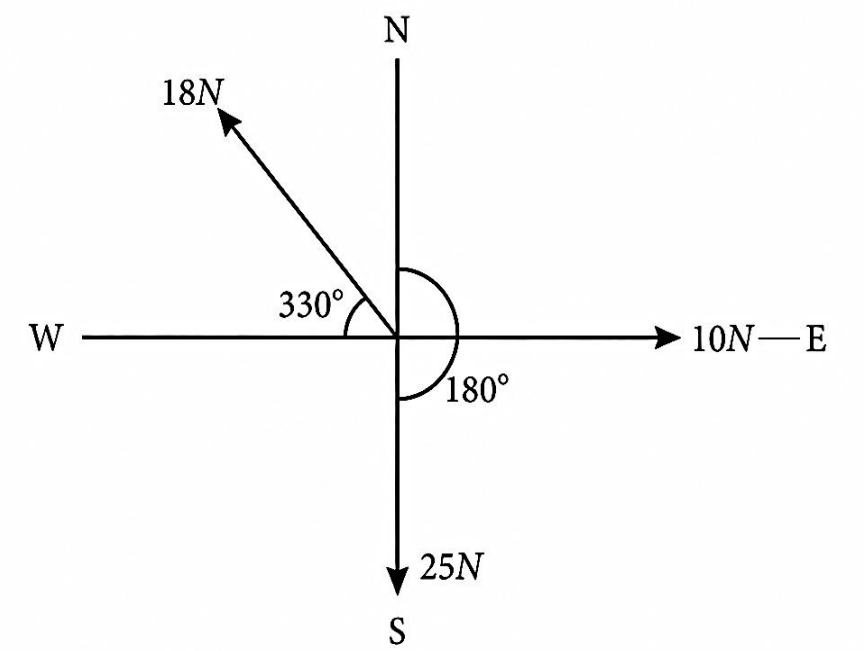
8

The forces are:

F_1 = (18 N, 330°).

F_2 = (10 N, 090°).

F_3 = (25 N, 180°).



F_1 = (−18 cos 60°, 18 sin 60°)

= (−18 × 0.5, 18 × 0.8660)

= (−9, 15.588).

F_2 = (10 sin 90°, 10 cos 90°)

= (10, 0).

F_3 = (0, −25).

The resultant force is:

R = (−9, 15.588) + (10, 0) + (0, −25)

= (1, −9.412).

Magnitude:

$|R| = \sqrt{[1^2 + (-9.412)^2]}$

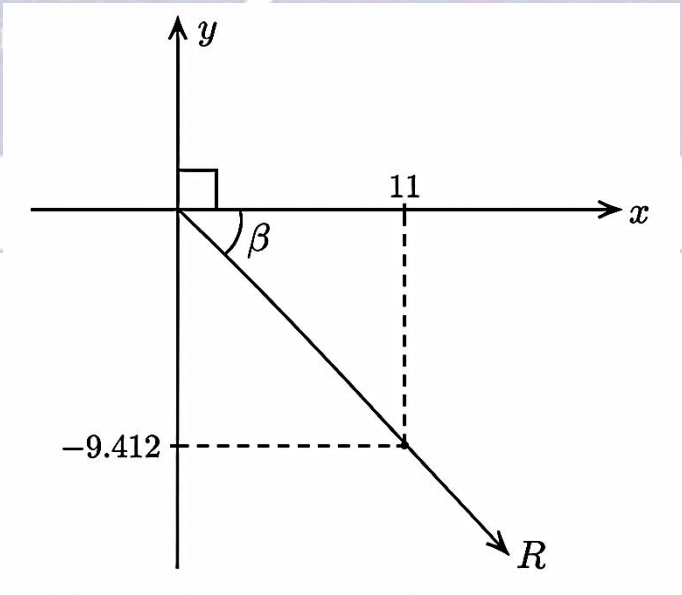
$= \sqrt{(1 + 88.585744)}$

$= \sqrt{89.585744}$

= 9.46497459

≈ 9.5 N, to one decimal place.

Direction of the resultant force:



tan β = 9.412.

The direction as 173.9°, approximately 174°.

Answer: 9.5 N on a bearing of 174°.

9(a)1/(1 − cos θ) + 1/(1 + cos θ)

Taking the LCM:

= [(1 + cos θ) + (1 − cos θ)]/[(1 − cos θ)(1 + cos θ)]

= 2/(1 − $\cos^2\theta$).

Recall:

$\sin^2\theta + \cos^2\theta = 1$.

Therefore:

$\sin^2\theta = 1 - \cos^2\theta$.

Substituting:

2/(1 − $\cos^2\theta$)

= 2/$\sin^2\theta$

= 2 $\text{cosec}^2\theta$.

Answer: 2 $\text{cosec}^2\theta$.

9(b)

$y = x^2(x - 3)$

$= x^3 - 3x^2$.

$dy/dx = 3x^2 - 6x$.

At stationary points:

dy/dx = 0.

$3x^2 - 6x = 0$.

3x(x − 2) = 0.

Therefore:

x = 0 or x = 2.

When x = 0:

$y = 0^3 - 3(0^2) = 0$.

When x = 2:

$y = 2^3 - 3(2^2) = -4$.

The stationary points are:

(0, 0) and (2, −4).

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Equation of the line joining the points:

$(y - y_1)/(x - x_1) = (y_2 - y_1)/(x_2 - x_1)$.

$(y - 0)/(x - 0) = (-4 - 0)/(2 - 0)$.

$y/x = -4/2$.

$2y = -4x$.

$2y + 4x = 0$.

Dividing through by 2:

$y + 2x = 0$.

Distance between the points:

$d = \sqrt{[(y_2 - y_1)^2 + (x_2 - x_1)^2]}$

$= \sqrt{[(-4 - 0)^2 + (2 - 0)^2]}$

$= \sqrt{(16 + 4)}$

$= \sqrt{20}$

$= 2\sqrt{5}$

$= 4.47213595$.

Distance = 4.4721 units.

10(a)(i)

$f(x) = (2x - 3)/[(x^2 - 1)(x + 2)]$.

For f(x) to be undefined:

$(x^2 - 1)(x + 2) = 0$.

$(x + 1)(x - 1)(x + 2) = 0$.

Therefore:

$x + 1 = 0 \Rightarrow x = -1$.

$x - 1 = 0 \Rightarrow x = 1$.

$x + 2 = 0 \Rightarrow x = -2$.

Answer: x = −1, 1 or −2.

10(a)(ii)

$(2x - 3)/[(x^2 - 1)(x + 2)]$

$= (2x - 3)/[(x + 1)(x - 1)(x + 2)]$.

Let:

$(2x - 3)/[(x + 1)(x - 1)(x + 2)]$

$= A/(x + 1) + B/(x - 1) + C/(x + 2)$.

Then:

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$2x - 3$

$= A(x - 1)(x + 2) + B(x + 1)(x + 2) + C(x + 1)(x - 1)$.

When $x = 1$:

$B = -1/6$.

When $x = -1$:

$A = 5/2$.

When $x = -2$:

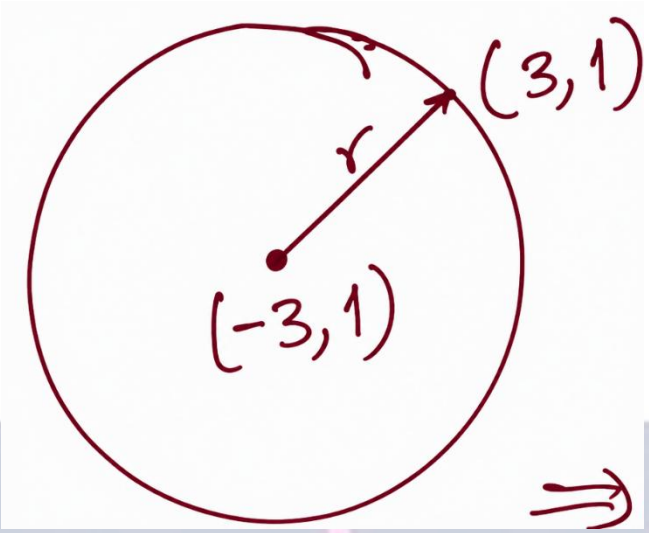
$C = -7/3$.

Substituting:

$(2x - 3)/[(x^2 - 1)(x + 2)]$

$= 5/[2(x + 1)] - 1/[6(x - 1)] - 7/[3(x + 2)]$.

10(b)



Centre = $(-3, 1)$.

Point on the circle = $(3, 1)$.

$r^2 = [3 - (-3)]^2 + (1 - 1)^2$

$= (3 + 3)^2 + 0^2$

$= 6^2$.

$r = 6$ units.

Equation of a circle:

$(x - a)^2 + (y - b)^2 = r^2$.

With $a = -3$ and $b = 1$:

$(x + 3)^2 + (y - 1)^2 = 6^2$.

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$x^2 + 6x + 9 + y^2 - 2y + 1 = 36.$

$x^2 + y^2 + 6x - 2y + 9 + 1 - 36 = 0.$

Answer: $x^2 + y^2 + 6x - 2y - 26 = 0.$

11(a)

$y = f(x) = \int(4x - x^2)\,dx$

$= 4x^2/2 - x^3/3 + C$

$= 2x^2 - x^3/3 + C.$

When $x = 3$, $f(x) = 21$.

$21 = 2(3^2) - 3^3/3 + C.$

$21 = 18 - 9 + C.$

$21 = 9 + C.$

$C = 21 - 9 = 12.$

Substituting:

$f(x) = 2x^2 - x^3/3 + 12.$

11(b)

For an arithmetic progression:

$T_2 = a + d.$

$T_4 = a + 3d.$

$T_8 = a + 7d.$

Since $a + d$, $a + 3d$ and $a + 7d$ are three terms of a geometric progression:

$(a + 3d)/(a + d) = (a + 7d)/(a + 3d).$

Therefore:

$(a + 3d)^2 = (a + d)(a + 7d).$

$a^2 + 6ad + 9d^2 = a^2 + 8ad + 7d^2.$

$2d^2 - 2ad = 0.$

The handwritten solution takes:

$d = a.$ … (1)

Also:

$T_3 + T_5 = 20.$

$(a + 2d) + (a + 4d) = 20.$

$2a + 6d = 20.$

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a + 3d = 10. … (2)

Substituting d = a:

a + 3a = 10.

4a = 10.

a = 10/4 = 5/2.

Similarly:

d = 5/2.

(i) First four terms

a, a + d, a + 2d, a + 3d.

Therefore:

5/2, 5, 15/2, 10, or 2½, 5, 7½, 10.

(ii) Sum of the first 10 terms

$S_{10} = (10/2)[2a + (10 - 1)d]$

$= 5[2(5/2) + 9(5/2)]$

$= 5(5 + 45/2)$

$= 275/2$

$= 137.5.$

Answer: $S_{10} = 137.5$.

PART II

12(a)

The ratio of students who passed to those who failed is:

4 : 1.

Total parts = 4 + 1 = 5.

Let p represent the probability of passing and q the probability of failing.

p = 4/5.

q = 1/5.

The binomial expansion is $(p + q)^7$.

Coefficients:

1, 7, 21, 35, 35, 21, 7, 1.

Thus:

$(p + q)^7$

$= p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 + 35p^3q^4 + 21p^2q^5 + 7pq^6 + q^7.$

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Probability that at least three students passed

$P(X \geq 3)$

$= p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 + 35p^3q^4.$

Alternatively:

$P(X \geq 3)$

$= 1 - (21p^2q^5 + 7pq^6 + q^7)$

$= 1 - [21(4/5)^2(1/5)^5 + 7(4/5)(1/5)^6 + (1/5)^7].$

$= 1 - [336/15,625 + 28/78,125 + 1/78,125]$

$= 1 - (0.021504 + 0.0003584 + 0.0000128)$

$= 1 - 0.0218752$

$= 0.9781248$

≈ 0.98, to two decimal places.

The first fraction should be 336/78,125. The correct probability from the stated formula is 0.995328, which rounds to 1.00 to two decimal places.

Probability that between three and six students failed

The next page uses:

$35p^3q^4 + 21p^2q^5$

$= 35(4/5)^3(1/5)^4 + 21(4/5)^2(1/5)^5$

$= (35 \times 64)/78,125 + (21 \times 16)/78,125$

$= 2,240/78,125 + 336/78,125$

$= 2,576/78,125$

$= 0.0329728$

≈ 0.03, to two decimal places.

12(b)

Probability of obtaining a six:

$p = 1/6.$

Probability of not obtaining a six:

$q = 1 - 1/6 = 5/6.$

For five throws:

$(p + q)^5$

$= p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5.$

Probability of obtaining a six exactly three times:

$P(X = 3) = 10p^3q^2$

$= 10(1/6)^3(5/6)^2$

$= (10 \times 25)/7,776$

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= 250/7,776

= 0.032150.

Answer: 0.03215.

13

Age (years)	Frequency, f	Class boundaries	Midpoint, x	fx
17–19	6	16.5–19.5	18.0	108
20–22	9	19.5–22.5	21.0	189
23–28	12	22.5–28.5	25.5	306
29–34	18	28.5–34.5	31.5	567
35–43	18	34.5–43.5	39.0	702
Total	63			1,872

Mean:

$\bar{x} = \Sigma fx/\Sigma f$

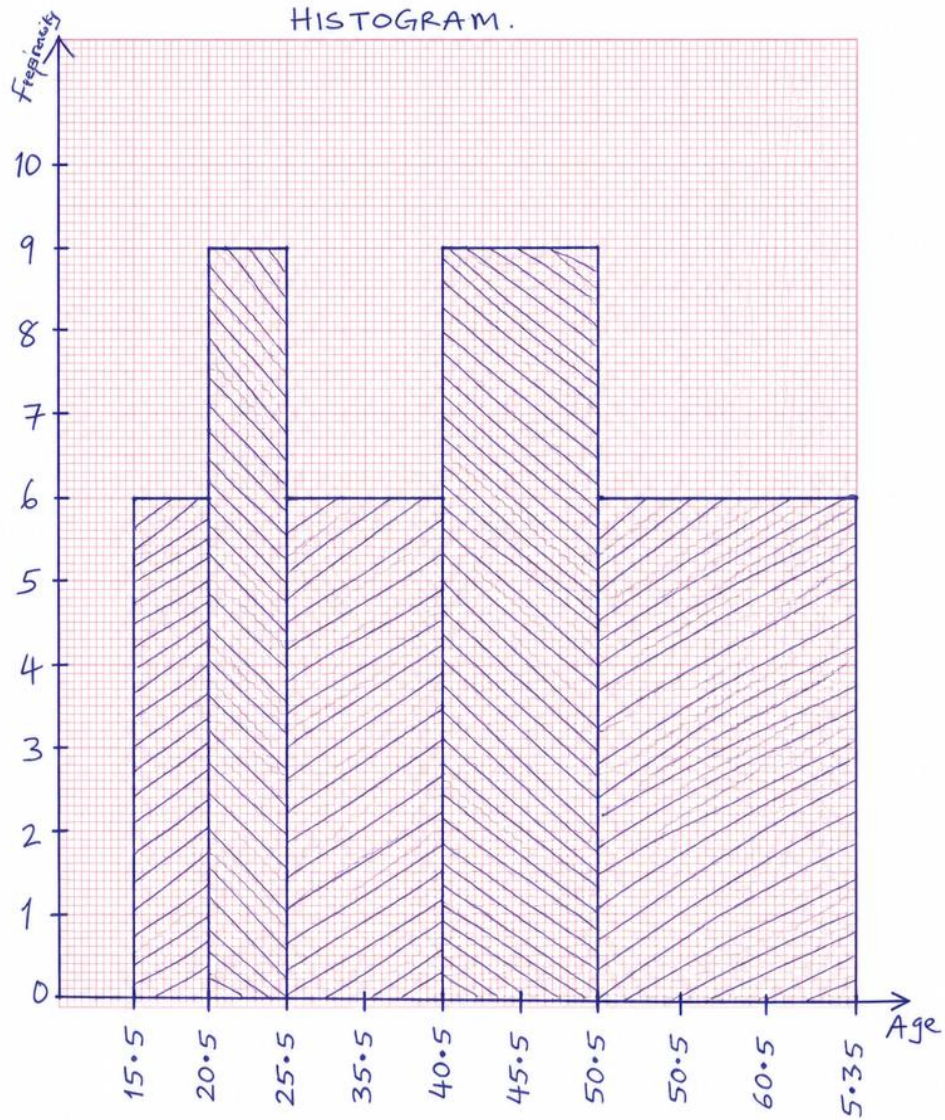
= 1,872/63

= 29.7142

≈ 29.71, to two decimal places.

Mean age of the patients = 29.71 years.

Working for the histogram:

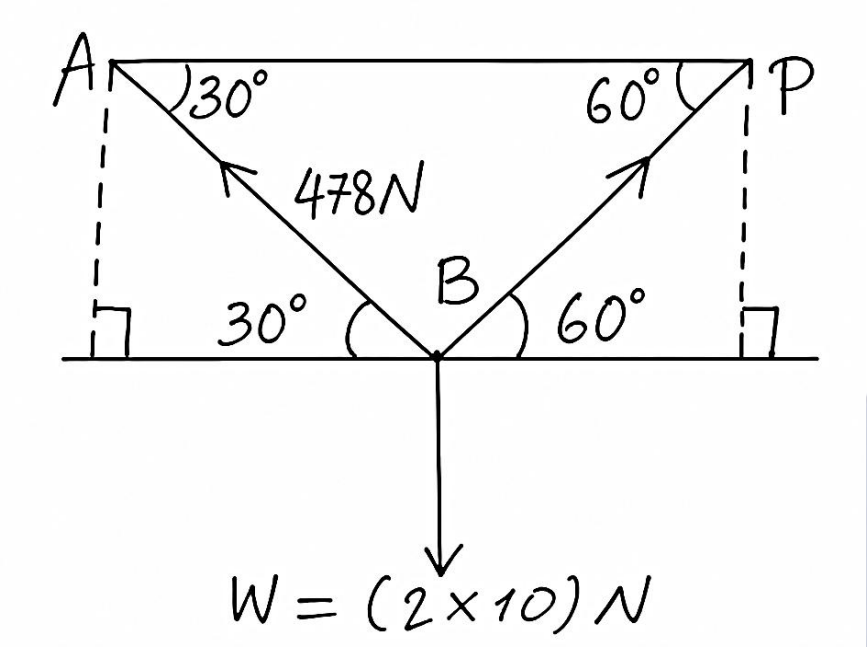


Class width	Area	Height
3	6 × 3 = 18	6
3	9 × 3 = 27	9
6	12 × 3 = 36	6
6	18 × 3 = 54	9
9	18 × 3 = 54	6

PART III

14(a)(i)

Force diagram, showing the 48 N tension, the 30° and 60° angles, and W = (q × 10) N:



14(a)(ii)

BD/sin 120° = 48/sin 150°.

Therefore:

BD = (48 sin 120°)/sin 150°

= (48 × 0.8660)/0.5000

= 41.568/0.5

= 83.136.

Tension = 83.14 N.

14(a)(iii)

48/sin 150° = 10q/sin 90°.

10q sin 150° = 48 sin 90°.

10q = (48 sin 90°)/sin 150°.

q = (48 sin 90°)/(10 sin 150°)

= 48/[10(0.5)]

= 48/5

= 9.6.

14(b)

u = 20 m/s.

g = 10 m/s².

v = 0.

Taking upward motion as positive, acceleration = −10 m/s².

Using:

v = u + at.

0 = 20 + (−10)t.

0 = 20 − 10t.

10t = 20.

t = 20/10 = 2.

Time = 2.00 seconds.

15(a)

m = 6i + 8j.

n = −8i + (7/3)j.

15(a)(i): Magnitude and direction of m

|m| = √(6² + 8²)

= √(36 + 64)

= √100

= 10.

tan θ = 8/6 = 4/3 = 1.3333.

θ = $\tan^{-1}$(1.3333)

≈ 53.12°.

Direction of m:

90° − 53.12° = 36.88°.

Magnitude and direction of m = (10, 036.88°).

Magnitude and direction of n

|n| = √[(-8)² + (7/3)²]

≈ √[64 + (2.333)²]

= √(64 + 5.442889)

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= √69.442889

≈ 8.333.

Therefore:

|n| ≈ 8.33.

Using the acute reference angle:

$\beta = \tan^{-1}(0.29162)$

= 16.259°

≈ 16.26°.

Direction of n:

270° + 16.26° = 286.26°.

Magnitude and direction of n = (8.33, 286.26°).

15(a)(ii): Angle between m and n

$m \cdot n = (6i + 8j) \cdot [-8i + (7/3)j]$

= −48 + 0 + 0 + 56/3

≈ −48 + 18.67

= −29.33.

Recall:

|m| = 10.

|n| = 8.33.

Using:

$m \cdot n = |m||n| \cos \theta$.

Therefore:

$\cos \theta = (m \cdot n)/(|m||n|)$

= −29.33/(10 × 8.33)

= −29.33/83.3

≈ −0.35174.

$\theta = \cos^{-1}(-0.35174)$

≈ 110.59°.

Angle between the vectors ≈ 111°.

15(b)

Vector PQ = Q − P

= (10, 4) − (−2, 3)

= (12, 1).

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Vector RS = S − R

= (4, 0) − (3, 12)

= (1, −12).

Their dot product is:

PQ · RS = (12 × 1) + [1 × (−12)]

= 12 − 12

= 0.

Since PQ · RS = 0, the angle between the vectors is 90°.

Hence, PQ is perpendicular to RS.

Recall that two non-zero vectors are perpendicular if their dot, or scalar, product is zero.

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2018 FURTHER MATHEMATICS ESSAY 2 — SECTION A

SECTION A

QUESTION 1

Solve the determinant equation:

$$(x-3)\begin{vmatrix} 2 & 2 \\ -4 & 6-x \end{vmatrix} + 4\begin{vmatrix} 5 & 2 \\ 2 & 6-x \end{vmatrix} + 3\begin{vmatrix} 5 & 2 \\ 2 & -4 \end{vmatrix} = -24$$

(x − 3)|2 2; −4 6−x| + 4|5 2; 2 6−x| + 3|5 2; 2 −4| = −24

(x − 3){2(6 − x) + 8} + 4{5(6 − x) − 4} + 3{5(−4) − 4} = −24

(x − 3){12 − 2x + 8} + 4{30 − 5x − 4} + 3{−20 − 4} = −24

(x − 3)(20 − 2x) + 4(26 − 5x) + 3(−24) = −24

20x − 2x² − 60 + 6x + 104 − 20x − 72 = −24

Collecting like terms:

−2x² + 6x − 28 = −24

2x² − 6x + 28 − 24 = 0

2x² − 6x + 4 = 0

Dividing through by 2:

x² − 3x + 2 = 0

x² − x − 2x + 2 = 0

x(x − 1) − 2(x − 1) = 0

(x − 2)(x − 1) = 0

x = 2 or x = 1

QUESTION 2

$\log_3 x - 3\log_x 3 + 2 = 0$

Let $\log_x 3 = F$. Since $\log_3 x = 1/\log_x 3$, this becomes $1/F - 3F + 2 = 0$.

$1 - 3F^2 + 2F = 0$

$3F^2 - 2F - 1 = 0$

Factorising:

3F(F − 1) + 1(F − 1) = 0

(3F + 1)(F − 1) = 0

Either 3F + 1 = 0, so F = −1/3, or F − 1 = 0, so F = 1.

Recall $F = \log_x 3$:

Either $\log_x 3 = -1/3$ or $\log_x 3 = 1$

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x^(−1/3) = 3 or $x^1 = 3$

$x = 3^{-3}$ or x = 3

x = 1/27 or x = 3

QUESTION 3

If u = x − 2, then x = u + 2.

Express $(x^3 + 5)/(x - 2)^4$ in terms of u:

$= ((u + 2)^3 + 5)/u^4$

$= ((u + 2)(u + 2)(u + 2) + 5)/u^4$

$= ((u + 2)(u^2 + 4u + 4) + 5)/u^4$

$= (u^3 + 6u^2 + 12u + 8 + 5)/u^4$

$= (u^3 + 6u^2 + 12u + 13)/u^4$

$= u^3/u^4 + 6u^2/u^4 + 12u/u^4 + 13/u^4$

$= 1/u + 6/u^2 + 12/u^3 + 13/u^4$

Recall u = x − 2. Hence, substituting:

$= 1/(x - 2) + 6/(x - 2)^2 + 12/(x - 2)^3 + 13/(x - 2)^4$

(The first term is hard to read in the photo, but 1/(x − 2) is what the working gives.)

QUESTION 4

$S_n = n/2 \{2a + (n - 1)d\}$

$S_{12} = 12/2 \{2a + (12 - 1)d\} = 168$

$6\{2a + 11d\} = 168$

Dividing both sides by 6:

2a + 11d = 28 ……(1)

Recall $T_3 = a + 2d = 7$ (given) ……(2)

Multiplying equation (2) by 2:

2a + 4d = 14 ……(3)

Subtract equation (3) from (1):

7d = 14

d = 14/7 = 2

Put d = 2 in equation (2):

a + 2(2) = 7

a + 4 = 7

a = 7 − 4 = 3

Common difference = 2; first term = 3

QUESTION 5

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Type (cooking oil)	A	B	C	D	E	F	G	H
X ranking	8	5	1	7	2	6	3	4
Y ranking	6	3	4	8	5	7	1	2
D = Rx – Ry	2	2	−3	−1	−3	−1	2	2
D^2	4	4	9	1	9	1	4	4

$\Sigma D^2 = 4 + 4 + 9 + 1 + 9 + 1 + 4 + 4 = 36$

$N = 8$

Applying Spearman's rank correlation coefficient formula:

$r_s = 1 - 6\Sigma D^2/(N(N^2 - 1))$

$= 1 - 6(36)/(8(8^2 - 1))$

$= 1 - 216/(8(64 - 1))$

$= 1 - 216/(8(63))$

$= 1 - 216/504$

$= 1 - 0.428571$

$= 0.571429$

$r_s = 0.5714$

QUESTION 6

(a)

Pr(Kunle solves a problem) = 1/3

Pr(Kunle does not solve the question) = 1 − 1/3 = 2/3

Pr(Tayo solves a question) = 1/5

Pr(Tayo does not solve the question) = 1 − 1/5 = 4/5

Pr(only one of them solves the question) = either Kunle solves and Tayo doesn't, or Tayo solves and Kunle doesn't

$= (1/3 \times 4/5) + (1/5 \times 2/3)$

$= 4/15 + 2/15$

$= 6/15$

$= 2/5$

(b) If there is no restriction, a committee of 8 can be selected from 10 persons in $^{10}C_8$ ways:

$^{10}C_8 = 10!/(8!\ 2!)$

$= (10 \times 9 \times 8!)/(8! \times 2 \times 1)$

$= (10 \times 9)/2$

$= 90/2$

This can be done in 45 ways.

QUESTION 7

$m = 3i - 2j$, $n = 2i + 3j$, $p = -i + 6j$

$4m + 2n - 3p$

$= 4(3i - 2j) + 2(2i + 3j) - 3(-i + 6j)$

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= 12i − 8j + 4i + 6j + 3i − 18j

= 19i − 20j

QUESTION 8

(a) SEE DIAGRAM (20 kg moving at 80 m/s and 30 kg moving at 50 m/s, both in the same direction)

Before collision, from the principle of conservation of linear momentum:

$m_1u_1 + m_2u_2 = (m_1 + m_2)V$ (where V = velocity after impact)

$20 \times 80 + 30 \times 50 = (20 + 30)V$

$1600 + 1500 = 50V$

$3100 = 50V$

$V = 3100/50 = 62$ m/s

(b) If after impact the bodies move in opposite directions:

Before collision, momentum = $m_1u_1 + m_2(-u_2) = (m_1 + m_2)V$

$20 \times 80 + 30(-50) = (20 + 30)V$

$1600 - 1500 = 50V$

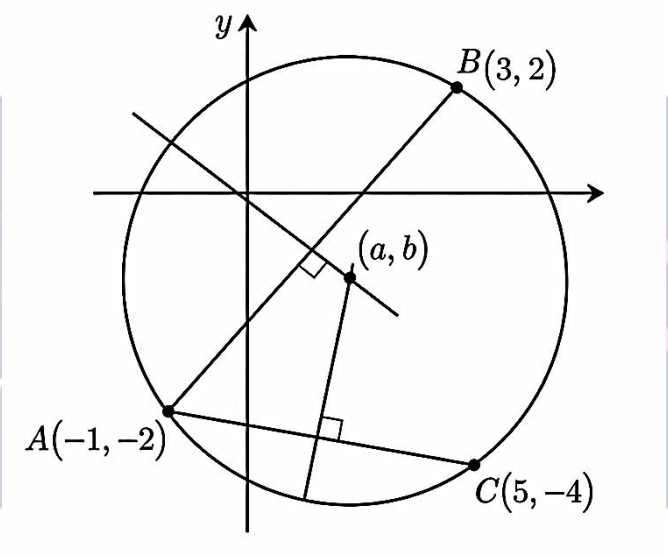
$100 = 50V$

$V = 100/50 = 2$ m/s

SECTION B

QUESTION 9

Let the points with coordinates (−1, −2), (3, 2) and (5, −4) be A, B and C respectively. Let the coordinates of the centre be (a, b).



(circle through A(−1, −2), B(3, 2), C(5, −4), with the perpendicular bisectors meeting at the centre (a, b))

Chord AB

Applying the two-point form $(y - y_1)/(y_2 - y_1) = (x - x_1)/(x_2 - x_1)$:

$(y - (-2))/(2 - (-2)) = (x - (-1))/(3 - (-1))$

$(y + 2)/4 = (x + 1)/4$

Cross multiplying:

$4(y + 2) = 4(x + 1)$

Dividing both sides by 4:

$y + 2 = x + 1$

$y - x = 1 - 2$

$y - x = -1$, or $x - y = 1$

The equation of the perpendicular L_1 through the midpoint of chord AB is of the form $x + y = c$.

Midpoint of AB = $(½(x_1 + x_2), ½(y_1 + y_2))$

$= (½(-1 + 3), ½(-2 + 2))$

$= (½(2), ½(0))$

$= (1, 0)$

Where $x = 1$, $y = 0$, so $c = 1 + 0 = 1$.

Therefore the equation of the perpendicular L_1 is $x + y = 1$.

Chord AC

$(y - (-2))/(-4 - (-2)) = (x - (-1))/(5 - (-1))$

$(y + 2)/(-2) = (x + 1)/6$

Cross multiplying:

$6(y + 2) = -2(x + 1)$

$6y + 12 = -2x - 2$

$6y + 2x = -2 - 12$

$6y + 2x = -14$

Dividing through by 2:

$3y + x = -7$, or $x + 3y = -7$

The equation of the perpendicular L_2 through the midpoint of chord AC is taken in the form $x + 3y = c$.

Midpoint of AC = $(½(-1 + 5), ½(-2 + (-4)))$

$= (½(4), ½(-6))$

$= (2, -3)$

Substituting in $x + 3y = c$:

$2 + 3(-3) = c$

$2 - 9 = c$

$c = -7$

Centre of the circle, using the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$

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For point (3, 2):

$9 + 4 + 6g + 4f + c = 0$

$6g + 4f + c = -13$ ……(1)

For point (−1, −2):

$1 + 4 - 2g - 4f + c = 0$

$-2g - 4f + c = -5$ ……(2)

For point (5, −4):

$25 + 16 + 10g - 8f + c = 0$

$10g - 8f + c = -41$ ……(3)

Adding equations (1) and (2):

$4g + 2c = -18$

Dividing through by 2:

$2g + c = -9$ ……(4)

Equating (2) and (3), multiplying (2) by 2:

$-4g - 8f + 2c = -10$ ……(5)

$10g - 8f + c = -41$ ……(3)

Subtracting (3) from (5):

$-14g + c = 31$ ……(6)

Equating (4) and (6), subtracting:

$2g + c = -9$

$-14g + c = 31$

$16g = -40$

$g = -40/16 = -5/2$

Put $g = -5/2$ in equation (4):

$2(-5/2) + c = -9$

$-5 + c = -9$

$c = -9 + 5 = -4$

Substituting for g and c in equation (1):

$6(-5/2) + 4f - 4 = -13$

$-15 + 4f - 4 = -13$

$4f - 19 = -13$

$4f = -13 + 19 = 6$

$f = 6/4 = 3/2$

Hence $f = 3/2$, $g = -5/2$, $c = -4$.

Hence the centre coordinates of the circle = $(5/2, -3/2)$

(b) Radius, $r = \sqrt{((3 - 5/2)^2 + (2 + 3/2)^2)}$

$= \sqrt{((1/2)^2 + (7/2)^2)}$

$= \sqrt{(1/4 + 49/4)}$

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= √(50/4)

= √(25 × 2/4)

Radius = 5√2/2 units

(c) Equation of the circle: $(x - a)^2 + (y - b)^2 = r^2$

$(x - 5/2)^2 + (y - (-3/2))^2 = 50/4$

$(x - 5/2)^2 + (y + 3/2)^2 = 50/4$

$x^2 - 5x + 25/4 + y^2 + 3y + 9/4 = 50/4$

$x^2 + y^2 - 5x + 3y + 34/4 = 50/4$

$x^2 + y^2 - 5x + 3y + 34/4 - 50/4 = 0$

$x^2 + y^2 - 5x + 3y - 16/4 = 0$

$x^2 + y^2 - 5x + 3y - 4 = 0$

Hence, required equation: $x^2 + y^2 - 5x + 3y - 4 = 0$

QUESTION 10

(a) $2^{(3y+2)} - 7(2^{(2y+2)}) - 31(2^y) - 8 = 0$

$2^{(3y)} \times 2^2 - 7(2^{(2y)} \times 2^2) - 31(2^y) - 8 = 0$

$2^{(3y)} \times 4 - 7(2^{(2y)} \times 4) - 31(2^y) - 8 = 0$

Let 2^y be represented by P:

$4(2^y)^3 - 7 \times 4(2^y)^2 - 31(2^y) - 8 = 0$

$4P^3 - 28P^2 - 31P - 8 = 0$

Solving the cubic equation:

$(P - 8)(2P + 1)(2P + 1) = 0$

Either $P - 8 = 0$, so $P = 8$,

or $2P + 1 = 0$, so $P = -1/2$ (repeated).

Recall $P = 2^y$:

Either $2^y = 8$, so $2^y = 2^3$,

or $2^y = -1/2$, which has no solution here.

Hence, $y = 3$

(b) $\int(\sqrt{(x^2 + 1)})\, x\, dx$

Let $u = x^2 + 1$

$du/dx = 2x$

$du = 2x\, dx$, so $\frac{1}{2}\, du = x\, dx$

$\int(\sqrt{(x^2 + 1)})\, x\, dx = \frac{1}{2} \int u^{(1/2)}\, du$

$= \frac{1}{2} (u^{(3/2)}/(3/2)) + c$

$= \frac{1}{3}\, u^{(3/2)} + c$

But $u = x^2 + 1$, so substituting for u:

$= \frac{1}{3} (x^2 + 1)^{(3/2)} + c$

QUESTION 11

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(a) Expansion of $(2 - ½x)^5$

Coefficients: 1, 5, 10, 10, 5, 1

$= 1(2^5)(-½x)^0 + 5(2^4)(-½x)^1 + 10(2^3)(-½x)^2 + 10(2^2)(-½x)^3 + 5(2^1)(-½x)^4 + 1(2^0)(-½x)^5$

$= 2^5 + 80(-½x) + 80(¼x^2) + 40(-⅛x^3) + 10(1/16\ x^4) + (-1/32\ x^5)$

$= 32 - 40x + 20x^2 - 5x^3 + (10/16)x^4 - x^5/32$

or $32 - 40x + 20x^2 - 5x^3 + (5/8)x^4 - x^5/32$

(a)(ii) Hence evaluate $(1.99)^5$ to 2 decimal places.

$(1.99)^5 = (2 - 0.01)^5$

If ½x = 0.01, then x = 0.02.

$= 32 - 40(0.02) + 20(0.02)^2 - 5(0.02)^3 + (5/8)(0.02)^4 - (0.02)^5/32$

$= 32 - 0.8 + 0.008 - 0.00004 + 0.0000001 + ...$

$= 31.2079601$

= 31.21 (2 decimal places)

(b) $f(x) = x^3 + gx^2 + rx + 9$

(x + 1) is a factor: if x + 1 = 0, then x = −1, so f(−1) = 0.

$f(-1) = (-1)^3 + g(-1)^2 + r(-1) + 9 = 0$

$-1 + g - r + 9 = 0$

$g - r + 8 = 0$

$g - r = -8$ ……(1)

If x + 2 = 0, then x = −2, so f(−2) = −17.

$f(-2) = (-2)^3 + g(-2)^2 + r(-2) + 9 = -17$

$-8 + 4g - 2r + 9 = -17$

$4g - 2r + 1 = -17$

$4g - 2r = -17 - 1 = -18$

Dividing through by 2:

$2g - r = -9$ ……(2)

Subtracting equation (2) from equation (1):

$(g - r) - (2g - r) = -8 - (-9)$

$-g = 1$

Hence g = −1

Put g = −1 in equation (1):

$-1 - r = -8$

$-r = -8 + 1 = -7$

r = 7

g = −1 and r = 7

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PART II

QUESTION 12

(a) Number of heads (x), frequency (f), and fx:

No. of heads (x)	Frequency (f)	fx
0	2	0
1	7	7
2	23	46
3	36	108
4	11	44
5	61	305
6	100	600
7	12	84
8	8	64
9	5	45
10	3	30
	Σf = 268	Σfx = 1333

Mean, $\bar{x} = \Sigma fx/\Sigma f = 1333/268$

= 4.97388…

Mean, $\bar{x}$ = 4.974 (3 decimal places)

(b) Pr(even number of heads) = (2 + 23 + 11 + 100 + 8 + 3)/268 = 145/268 (as written on the page)

= 0.5410447…

= 0.541 (3 decimal places)

(c) Pr(odd number of heads) = (7 + 36 + 61 + 12 + 5)/268

= 121/268

= 0.45149

= 0.451 (3 decimal places)

QUESTION 13

Pr(Ali gains admission) = 2/3, so Pr(Ali will not gain admission) = 1 − 2/3 = 1/3

Pr(Baba gains admission) = 3/4, so Pr(Baba will not gain admission) = 1 − 3/4 = 1/4

Pr(Kitty gains admission) = 4/5, so Pr(Kitty will not gain admission) = 1 − 4/5 = 1/5

(a) Pr(only Kitty and Baba gain admission) = 4/5 × 3/4 × 1/3 = 1/5

(b) Pr(none of them will gain admission) = 1/3 × 1/4 × 1/5 = 1/60

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(c) Pr(at most two of them gain admission) = Pr(none will gain admission) or Pr(only one of them will gain admission) or Pr(only two of them will gain admission)

= (1/3 × 1/4 × 1/5)

(2/3 × 1/4 × 1/5) + (1/3 × 3/4 × 1/5) + (1/3 × 1/4 × 4/5)

(2/3 × 3/4 × 1/5) + (2/3 × 1/4 × 4/5) + (1/3 × 3/4 × 4/5)

= 1/60 + 2/60 + 3/60 + 4/60 + 6/60 + 8/60 + 12/60

= (1 + 2 + 3 + 4 + 6 + 8 + 12)/60

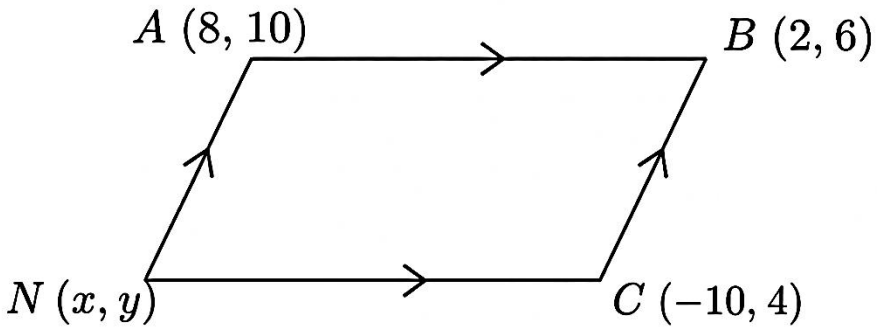
= 36/60

Pr(at most two gain admission) = 3/5

PART III

QUESTION 14

A(8, −10), B(2, 6), C(−10, 4), N(x, y)



(points A, B, C and N, with the vectors drawn)

(a) AN = (x, y) − (8, −10) = (x − 8, y + 10)

BC = (−10, 4) − (2, 6) = (−12, −2)

Since AN = BC:

(x − 8, y + 10) = (−12, −2)

x − 8 = −12, so x = −12 + 8 = −4

y + 10 = −2, so y = −2 − 10 = −12

Hence, ON = −4i − 12j

(b) AN = (−4, −12) − (8, −10) = (−4 − 8, −12 + 10) = (−12, −2)

|AN| = √((−12)² + (−2)²) = √(144 + 4) = √148 units

AB = (2, 6) − (8, −10) = (2 − 8, 6 + 10) = (−6, 16)

|AB| = √(6² + 16²) = √(36 + 256) = √292 units

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(c) $\cos\theta = (AN \cdot AB)/(|AN||AB|)$

$= ((-12)(-6) + (-2)(16))/(\sqrt{148} \times \sqrt{292})$

$= (72 - 32)/(\sqrt{148} \times \sqrt{292})$

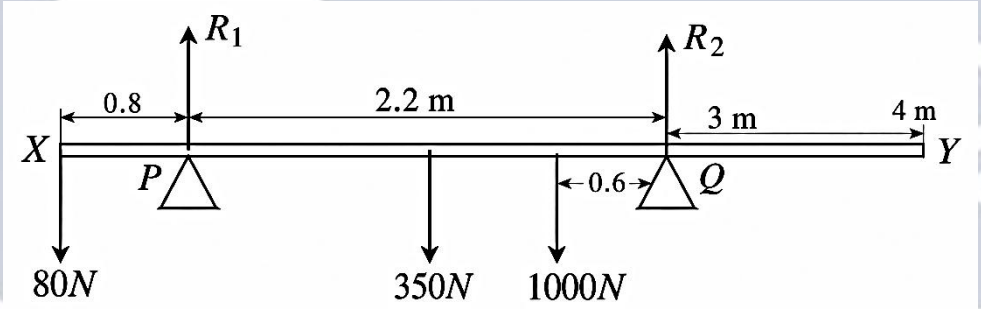
$= 40/\sqrt{43,216}$

$= 40/207.88458$

$= 0.19241$

$\theta = \cos^{-1}(0.19241) = 78.91°$

QUESTION 15



(beam XY, 4 m long, resting on supports P and Q with reactions R_1 at P and R_2 at Q. Loads: 80 N at X, 350 N and 1000 N on the beam. |XP| = 0.8 m, |PQ| = 2.2 m)

Taking moments about P (anticlockwise moment = clockwise moment):

$80 \times 0.8 + R_2 \times 2.2 = 350(1.2) + 1000(1.6)$

$64.0 + 2.2R_2 = 420.0 + 1600.0$

$2.2R_2 = 2020.0 - 64.0$

$2.2R_2 = 1956.0$

$R_2 = 1956.0/2.2 = 889.0909$

But sum of upward forces = sum of downward forces:

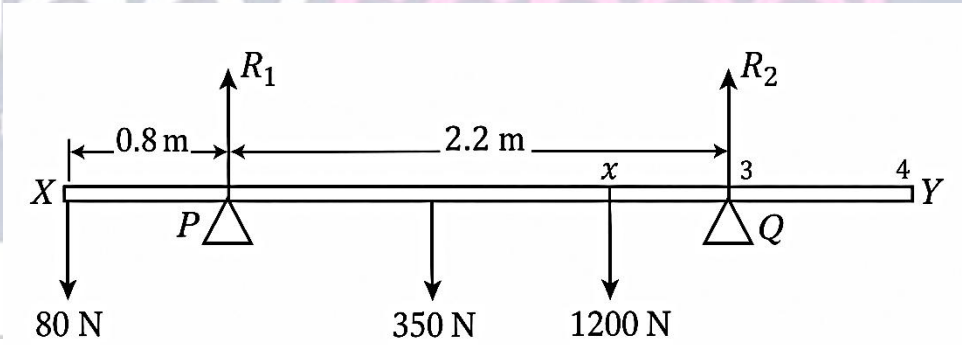
$R_1 + R_2 = 80 + 350 + 1000$

$R_1 + 889.09 = 1430$

$R_1 = 1430 - 889.09 = 540.91$

Hence, reactions at P and Q are 540.91 N and 889.09 N respectively.

(b



(the same beam with a 1200 N load at a point X between P and Q)

Taking moments of all forces about P (anticlockwise moments = clockwise moments):

$80 \times 0.8 + R_2 \times 2.2 = R_1 \times 0 + 350(1.2) + 1200x$

$64 + 2.2R_2 = 0 + 420 + 1200x$

$64 + 2.2(889.09) = 420 + 1200x$

$64 + 1955.998 = 420 + 1200x$

$2019.998 = 420 + 1200x$

$2019.998 - 420 = 1200x$

$1599.998 = 1200x$

$x = 1599.998/1200 = 1.3333$

$|PX| = 1.33$ m

Distance of the 1200 N from Q = $|PQ| - |PX|$

$= 2.2 - 1.3$

$= 0.9$ m

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WASSCE JUNE 2019 — FURTHER MATHEMATICS 2 (ESSAY)

(Section A)

Question 1

625(3x/4-1) + 125(x-1) / 5^(3x-2)

= $[5^{4(3x/4-1)} + 5^{3(x-1)}]$ / 5^(3x-2)

= [5^(3x-4) + 5^(3x-3)] / $5^{(3x)} \cdot 5^{-2}$

Factorising: $5^{(3x)}(5^{-4}+5^{-3})$ / $5^{(3x)}(5^{-2})$

= $5^{-4}+5^{-3} \div 5^{-2}$

= 1/625 + 1/125 ÷ 1/25

= (1+5)/625 × 25/1

= 150/625

= 6/25

Question 2

(a) Let the coordinates of the point be P(x,y)

P(x,y) = {3(-2)-2(7), 3(7)-2(9)} / {3-2, 7-2}

= {-6-14, 21-18} / 1

= (-20, 31)

∴ The coordinates = (-20, 31)

(b) 2/(1-√2) - 2/(2+√2)

Rationalising:

= [2/(1-√2) × (1+√2)/(1+√2)] - [2/(2+√2) × (2-√2)/(2-√2)]

= (2+2√2)/(1-2) - (4-2√2)/(4-2)

= (2+2√2)/(-1) - (4-2√2)/2

By L.C.M:

= [2(2+2√2) - (-1)(4-2√2)] / 2(-1)

= (4+4√2) - (4-2√2) / -2

= (2+2√2) / -1

Question 3

-3, -1, 1, 1, ... d = -1-(-3) = -1+3 = 2

Sum of A.P: Sn = n/2{2a+(n-1)d} = 165

165 = n/2{2(-3)+(n-1)2}

165 = n/2{-6+2n-2}

165 = n/2{2n-8}

$165 = n^2-4n$

$n^2-4n-165 = 0$

Solving by factorisation: (n-15)(n+11) = 0

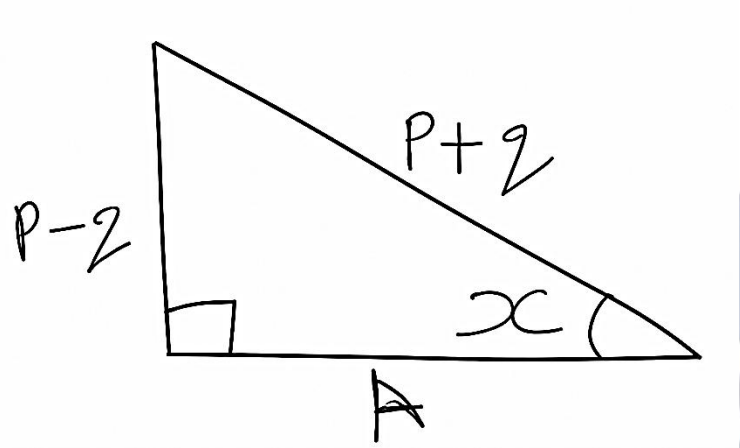
Either n-15=0 ⇒ n=15, or n+11=0 ⇒ n=-11

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Since n cannot be negative, n=15

Hence, the number of terms = 15

Question 4



Sin x = (p-q)/(p+q) = Opposite/Hypotenuse

Let the adjacent side be A

By Pythagoras: $A^2 = (p+q)^2-(p-q)^2$

$= \{(p+q)+(p-q)\}\{(p+q)-(p-q)\}$

$A^2 = (2p)(2q) = 4pq$

$\therefore A = \sqrt{4pq} = 2\sqrt{pq}$

Now tan x = Opposite/Adjacent = $(p-q)/2\sqrt{pq}$

Hence, $1+\tan^2 x = 1 + [(p-q)/2\sqrt{pq}]^2$

$= 1 + (p^2-2pq+q^2)/4pq$

Collecting like terms:

$= (4pq+p^2-2pq+q^2)/4pq$

$= (p^2+2pq+q^2)/4pq$

By L.C.M.

Question 5

Let the angle between the forces be x

Forces are 10N and 16N

$\cos\theta = (10^2+16^2-14^2) / (2\times10\times16)$

$= (100+256-196) / 320$

$= 160/320$

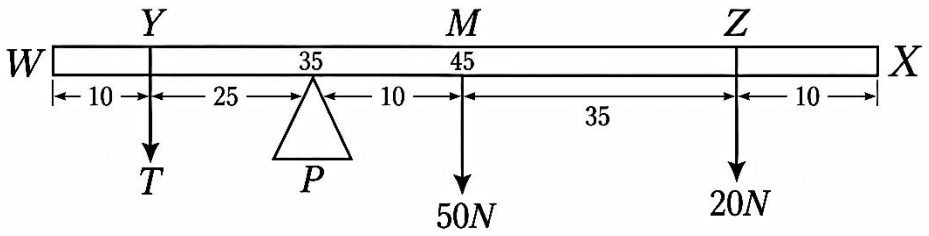
$\cos\theta = 0.5$

$\theta = \cos^{-1}(0.5) = 60°$

Now $x = 180-\theta = 180-60 = 120°$

Hence, angle between the forces = 120°

Question 6



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Centre of gravity = 45cm from M (5m)

Taking moments about pivot

By principle of moments: Anticlockwise moments = clockwise moments for equilibrium

7×25 = 50×10 + 20×45

25T = 500+900

25T = 1400

T = 1400/25 = 56

∴ T = 56N

Question 7

Assumed Mean, A = 13

Age	Class Mark (x)	Frequency (f)	d = x-A	fd
1-5	3	18	-10	-180
6-10	8	13	-5	-60
11-15	13	25	0	0
16-20	18	20	5	100
21-25	23	15	10	150
26-30	28	10	15	150
Total		Σf=100		Σfd=185

Mean, $\bar{x} = A + \Sigma fd/\Sigma f$

$\bar{x} = 13 + 185/100$

= 13+1.85

Mean, $\bar{x}$ = 14.85 years

Question 8

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Pr(a sum of 7) = 6/36

Pr(a sum of 4) = 3/36

Pr(a sum of 7 and a sum of 4) = 6/36 × 3/36 = 1/72

(PART II)

Question 9

(a) y = (7-6)/(2x-6); y = 7x/(2x-6)

Equations: y = (2x-6)/x

Cross-multiplying:

$-2x^2+4x-6 = 0$

$2x^2-4x+6 = 0$

Question 10

(a) $x^2-px+1 = 0$

For equality of roots: $b^2=4ac$

From the above equation (quadratic in x): a=1, b=-p, c=1

Recall for equality of roots: $b^2=4ac$

$(-p)^2 = 4(1)(1)$

$p^2 = 4(4)(1)$

$p^2 = 16$

$p = \pm\sqrt{16} = \pm4$

$\therefore p = 4 \text{ or } -4$

(b)(i) $(1+3x)^6$

Coefficients = 1,6,15,20,15,6,1

$1^6+6(1)^5(3x)^1+15(1)^4(3x)^2+20(1)^3(3x)^3+15(1)^2(3x)^4+6(1)(3x)^5+1(3x)^6$

$= 1+18x+135x^2+540x^3+1215x^4+1458x^5+729x^6$

(ii) $(1.03)^6 = (1+0.03)^6$

But $1+3x = 1.03$

$\Rightarrow x = 0.03/3 = 0.01$

Hence, we have $(1.03)^6 = (1+0.01)^6$

$=$

$1+18(0.01)+135(0.01)^2+540(0.01)^3+1215(0.01)^4+1458(0.01)^5+729(0.01)^6$

$= 1+0.18+0.0135+0.00054+0.0000121+0.0000001458+0.000000000729$

$= 1.1940525$

$\therefore (1.03)^6 \approx 1.194$ (4 significant figures)

Question 11

(a) M = (0, 7/3); N = (2/5, -1/3)

(b) MN = (0, 7/3)(2, -1/5, 3)

= (35/1, 21/-6)

Hence, image of P(2,-3) = (35, 21/1, -6)(2,-3)

= (70-63, 2+18)

= (7, 20)

Hence, coordinates of P' = (7, 20)

(c) Let the coordinates of Q be P(x,y)

From equation (1): 2y-y = 2x-2

Substituting in equation (1):

$5x+3(2x-2) = 4$

$5x+6x-6 = 4$

$11x = 4+6$

$11x = 10$

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x = 10/11

Substituting for x and y and solving:

y-2 = 3/5×10/11-2

= 20/11-2

= (20-22)/11

= -2/11

Hence, coordinates of Q = (10/11, -2/11)

Question 12 (PART III)

(a) p = 3/5 and q = 1-3/5 = 2/5

Pr(at least 2) = Pr(x≥2)

Pr(x≥2) = 1-Pr(x<2)

= 1-[Pr(x=0)+Pr(x=1)]

= 1-[$^{10}C_0(2/5)^{10}$ + $^{10}C_1(3/5)^1(2/5)^9$]

= 1-0.1673 (4 decimal places)

= 0.9536 (4 decimal places)

(b) Pr(x=6) = $^{10}C_6(3/5)^6(2/5)^4$

= 252(0.07776)(0.01024)

= 0.2006588

≈ 0.2007 (4 decimal places)

(c) Pr(x≤2) = Pr(x=0)+Pr(x=1)+Pr(x=2)

= $^{10}C_0(2/5)^{10}$ + $^{10}C_1(3/5)^1(2/5)^9$ + $^{10}C_2(3/5)^2(2/5)^8$

= 0.0060466176+0.040310784+0.120132352

= 0.167289953

≈ 0.1673 (4 decimal places)

Question 13

(a) Let performance in Entrance Exam be x; performance in School Exam be y

Student	x	y	Rx	Ry	D = Rx-Ry	D^2
A	11	5	7	9	-2	4
B	12	10	6	4	2	4
C	8	9	9	5	4	16
D	13	7	5	7	-2	4
E	8	4	10	10	0	0
F	15	8	3	6	-3	9
G	10	6	8	8	0	0
H	14	14	1	1	0	0
I	17	11	2	3	-1	1
J	16	12	4	2	2	4
					ΣD^2=50	N=10

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(b) Applying Spearman's Rank Correlation Coefficient:

$Rs = 1 - 6\Sigma D^2/N(N^2-1)$

$= 1 - 6(50)/10(99)$

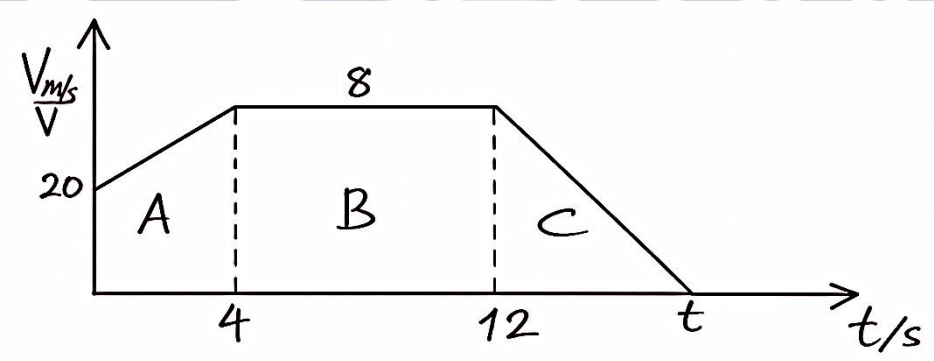
$= 1 - 300/990$

$= 1 - 0.303030$

$\approx 0.69697 \approx 0.697$

From the result above (correlation coefficient), it could be deduced that there is a positive correlation between the students' performance in the entrance examination and their school performance.

Question 14



(a) U = 20m/s, a = 2½ m/s², t = 4s

(b) Areas of A, B and C from the Sketch:

Total distance of the journey

$= ½(20+30)\times1 + (30\times8) + ½(240+135)$

Let the retardation be r

a = 2½ : 5½ = 3:4

$\Rightarrow r = 5½ \times ¾ = 20/6 = 10/3$

Again, 5½ : 30/t

$5/2 \times t = 30$

Cross multiplying: $5t\times4 = 60\times3$

$20t = 180$

$t = 180/20 = 9$ seconds

But velocity after 4 seconds $= 20 + 5/2\times4$

$= 20+10$

$= 30$ m/s

∴ Final velocity = 30 m/s

$= ½(200) + 240 + ½(270)$

$= 100+240+135$

= 475

∴ Total distance = 475m

Question 15

(a) M = i-j; N = 2i+3j

2M+N-r = 0

2(i-j) + 2i+3j - r = 0

2i-2j+2i+3j-r = 0

Hence, r = 4i+j

$|r| = \sqrt{(4^2+1^2)} = \sqrt{(16+1)} = \sqrt{17}$

∴ $|r| = \sqrt{17} = 4.1231$

(b)(i) $S = 3t - t^3/3 + 9$

$ds/dt = 3t^{1-1} - 3t^{3-1}/3$

$ds/dt = 3-t^2$

$ds/dt = 3-t^2 = 0$

$\Rightarrow t^2 = 3$

∴ $t = \sqrt{3} = 1.7321$ seconds

(ii) Distance travelled when the particle is at rest:

$S = 3t - t^3/3 + 9$

$= 3(1.7321) - (1.7321)^3/3 + 9$

$= 5.1963 - 5.1965195167/3 + 9$

$= 5.1963 - 1.7321983891 + 9$

$= 12.4641016$

∴ Distance travelled = 12.46410 metres

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2020 — WASSCE JUNE FURTHER MATHEMATICS 2 (ESSAY)

ANSWERS / SOLUTIONS

SECTION A

1(a)

Let x be the inverse of −1, such that:

−1 * x = x * −1 = 0.

Using the given operation:

−1 * x = −1 + x − (−½)x.

Recall that the identity element is 0.

Therefore:

−1 + x + x/2 = 0.

Multiplying through by 2:

−2 + 2x + x = 0.

−2 + 3x = 0.

3x = 2.

x = 2/3.

1(b)

m * 7 = m + 7 − 7m/2.

Taking the LCM:

m * 7 = (2m + 14 − 7m)/2

= (14 − 5m)/2. … (i)

Similarly:

m * 5 = m + 5 − 5m/2

= (2m + 10 − 5m)/2

= (10 − 3m)/2. … (ii)

Equating (i) and (ii):

(14 − 5m)/2 = (10 − 3m)/2.

Multiplying through by 2:

14 − 5m = 10 − 3m.

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14 − 10 = 5m − 3m.

4 = 2m.

m = 4/2 = 2.

Solution set: {2}.

2(a)

p: y → 2y + 3.

q: y → y^2 − 2.

Substituting p into q:

$(q \circ p)(y) = (2y + 3)^2 - 2$

$= (2y + 3)(2y + 3) - 2$

$= 4y^2 + 6y + 6y + 9 - 2$

$= 4y^2 + 12y + 7$.

Answer: $4y^2 + 12y + 7$.

2(b)

{1, 7}, {1, 3}, {1, 8}, {1, 2}, {7, 3}, {7, 8}, {7, 2}, {3, 8}, {3, 2}.

It then calculates:

$^9C_4 = {}^9P_4/4!$

$= (9 \times 8 \times 7 \times 6)/(4 \times 3 \times 2 \times 1)$

= 126 ways.

Hence, the number of four-digit odd numbers greater than 4,000 is written as:

4! + 126

$= 4 \times 3 \times 2 \times 1 + 126$

= 24 + 126

= 150.

Answer written: 150.

3

Evaluate:

$\int_1^3 (3x - 2)^5 \, dx$.

Let u = 3x − 2.

Then:

du = 3 dx.

dx = du/3.

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Therefore, an antiderivative is:

$(1/3)\int u^5\,du$

$= (1/3)(u^6/6)$

$= (3x - 2)^6/18.$

Evaluating between x = 1 and x = 3:

$[(3x - 2)^6/18]_1^3$

$= [3(3) - 2]^6/18 - [3(1) - 2]^6/18$

$= (9 - 2)^6/18 - (3 - 2)^6/18$

$= 7^6/18 - 1^6/18$

$= (117{,}649 - 1)/18$

$= 117{,}648/18.$

= 6,536.05556

≈ 6,536.1.

Answer written: 6,536.1.

Arithmetic correction: 117,648 ÷ 18 = 6,536 exactly.

4

$(3x + 1)/[(x - 1)(x + 1)]$

$= Q/(x - 1) + R/(x + 1).$

Multiplying through by (x − 1)(x + 1):

$3x + 1 = Q(x + 1) + R(x - 1).$

When x = −1:

$3(-1) + 1 = Q(-1 + 1) + R(-1 - 1).$

$-3 + 1 = -2R.$

$R = -2/2 = -1.$

When x = 1:

$3(1) + 1 = Q(1 + 1) + R(1 - 1).$

$4 = 2Q.$

$Q = 4/2 = 2.$

Answers written: Q = 2 and R = −1.

Sign correction: −2 = −2R gives R = 1. The correct decomposition is 2/(x − 1) + 1/(x + 1).

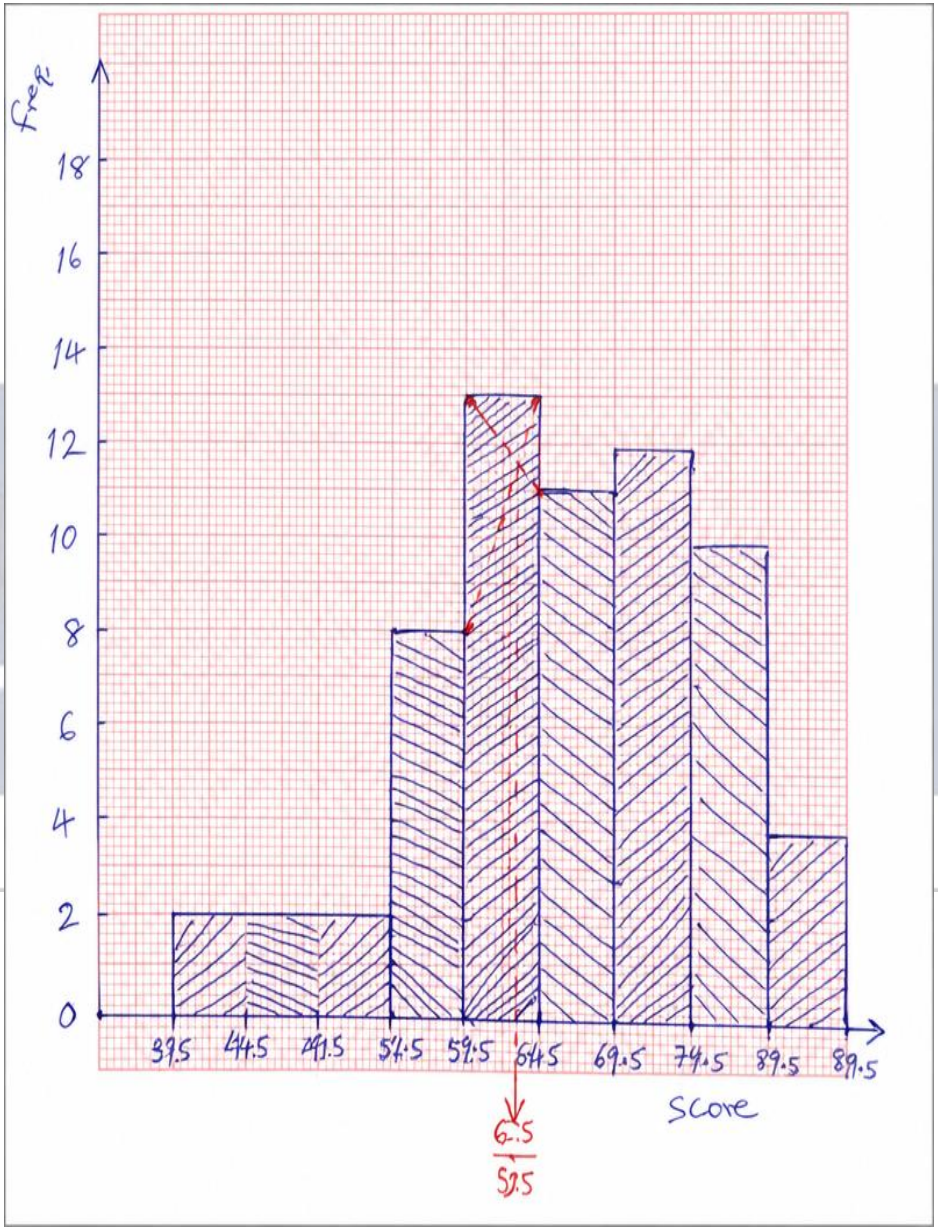
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5(a)

Mark	Frequency	Class boundaries
10–19	2	9.5–19.5
20–29	2	19.5–29.5
30–39	2	29.5–39.5
40–49	8	39.5–49.5
50–59	13	49.5–59.5
60–69	11	59.5–69.5
70–79	12	69.5–79.5
80–89	10	79.5–89.5
90–99	4	89.5–99.5

Histogram:



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5(b)From the histogram:

Modal score = 56.5%.

6(a)Red = 10.

Green = 8.

Total = 18.

Probability of one red and one green without replacement:

= (10/18) × (8/17)

= 40/153

= 0.2614379.

Answer written: 40/153 ≈ 0.2614.

Note: this calculation covers red first, then green. If either order is allowed, the probability of one of each is 80/153 ≈ 0.5229.

6(b)

Probability that a bulb is defective:

p = 20/100 = 0.2.

Probability that a bulb is non-defective:

q = 1 − 0.2 = 0.8.

Probability of between two and five defective bulbs is taken as:

P(X = 3) + P(X = 4).

Therefore:

$P = {}^{12}C_3(0.2)^3(0.8)^9 + {}^{12}C_4(0.2)^4(0.8)^8$.

$= (12 \times 11 \times 10)/(3 \times 2 \times 1)^3(0.8)^9$

$(12 \times 11 \times 10 \times 9)/(4 \times 3 \times 2 \times 1)^4(0.8)^8$.

= 220(0.008)(0.134217728)

495(0.0016)(0.16777216).

= 220(0.001073741824) + 495(0.000268435456).

= 0.23622320128 + 0.13287555072.

= 0.369098752.

Answer: 0.3691.

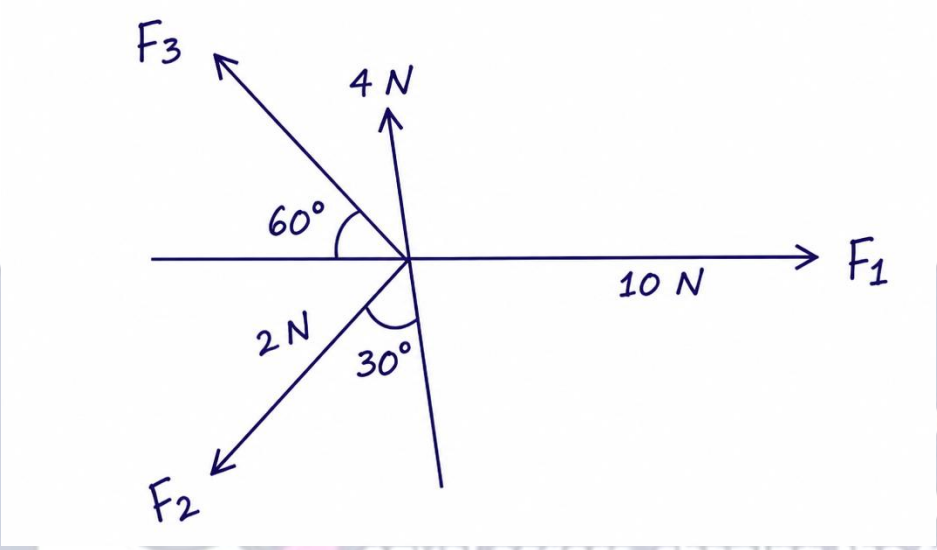
This uses “between two and five” exclusively: three or four defective bulbs.

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7

Force diagram:



Resolving the forces into horizontal and vertical components:

$F_1 = (10, 0)$.

$F_2 = (-20 \cos 60°, -20 \sin 60°)$

$= [-20(0.5), -20(0.866)]$

$= (-10, -17.32)$.

$F_3 = (-4 \cos 60°, 4 \sin 60°)$

$= [-4(0.5), 4(0.866)]$

$= (-2, 3.464)$.

Using the rounded value 3.46:

$R = (10, 0) + (-10, -17.32) + (-2, 3.46)$

$= (-2, -13.86)$.

Magnitude:

$|R| = \sqrt{[(-2)^2 + (-13.86)^2]}$

$= \sqrt{(4 + 192.0996)}$

$= \sqrt{196.0996}$

$= 14.00355669$.

Resultant force = 14.004 N.

---

8 $W = 8i + 3j = (8, 3)$.

$X = 6i - 5j = (6, -5)$.

$Y = 2i + 3j = (2, 3)$.

First:

W + X − 2Y

= (8, 3) + (6, −5) − 2(2, 3)

= (14 − 4, −2 − 6)

= (10, −8)

= 10i − 8j.

The required vector has magnitude 41 in this direction.

Therefore:

$Z = 41(10i - 8j)/\sqrt{[10^2 + (-8)^2]}$

$= 41(10i - 8j)/\sqrt{(100 + 64)}$

$= 41(10i - 8j)/\sqrt{164}$

$= 41(10i - 8j)/(2\sqrt{41})$.

Rationalising:

$Z = [41(10i - 8j)/(2\sqrt{41})] \times (\sqrt{41}/\sqrt{41})$

$= [41 \times 2(5i - 4j)\sqrt{41}]/(2 \times 41)$.

$Z = \sqrt{41}(5i - 4j)$.

SECTION B

9(a)

$g(x) = 2x^3 + 11x^2 - x - 30$.

Given that x + 2 is a factor, divide by x + 2:

$2x^3 + 11x^2 - x - 30$

$= (x + 2)(2x^2 + 7x - 15)$.

Factorising the quadratic:

$2x^2 + 7x - 15 = (2x - 3)(x + 5)$.

Therefore:

$g(x) = (x + 2)(2x - 3)(x + 5)$.

For g(x) = 0:

$x + 2 = 0 \Rightarrow x = -2$.

$2x - 3 = 0 \Rightarrow x = 3/2$.

$x + 5 = 0 \Rightarrow x = -5$.

Answers: x = −2, 3/2 or −5.

9(b)

$3(2^x) + 3^(y - 2) = 25$. … (1)

$2^x - 3^(y + 1) = -19$. … (2)

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Rewrite:

$3(2^x) + (3^y)/9 = 25$. … (3)

$2^x - 3(3^y) = -19$. … (4)

Let:

$V = 2^x$ and $W = 3^y$.

Then:

$3V + W/9 = 25$. … (5)

$V - 3W = -19$. … (6)

Multiplying equation (6) by 3:

$3V - 9W = -57$.

Subtracting this from equation (5):

$W/9 + 9W = 82$.

$(82/9)W = 82$.

$W = 82 \times 9/82 = 9$.

Substitute W = 9 into equation (6):

$V - 3(9) = -19$.

$V - 27 = -19$.

$V = 8$.

Since:

$2^x = 8 = 2^3$, $x = 3$.

$3^y = 9 = 3^2$, $y = 2$.

Answer: x = 3 and y = 2.

10(a)

$y = x^2(1 + x)^{(3/2)}$.

Using the product rule:

$dy/dx = 2x(1 + x)^{(3/2)} + x^2 \times (3/2)(1 + x)^{(1/2)}$.

Factorising:

$dy/dx = (1 + x)^{(1/2)}[2x(1 + x) + (3/2)x^2]$

$= (1 + x)^{(1/2)}[2x + 2x^2 + (3/2)x^2]$

$= (1 + x)^{(1/2)}[(4x + 7x^2)/2]$.

The final handwritten line is:

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dy/dx = ½(1 + x)^(1/2)(4 + 7x).

Correction: an x is missing from that final line. The derivative is (x/2)√(1 + x)(4 + 7x).

10(b)

The centre lies on:

2y − x = 3.

Let the centre be (a, b).

Then:

2b − a = 3.

b = (3 + a)/2.

Therefore, the centre is:

[a, (3 + a)/2].

The general equation of a circle is:

$x^2 + y^2 + 2gx + 2fy + c = 0$.

At the point (2, 3):

$2^2 + 3^2 + 2(2)g + 2(3)f + c = 0$.

$4 + 9 + 4g + 6f + c = 0$.

4g + 6f + c = −13. … (1)

At the point (6, 7):

$6^2 + 7^2 + 2(6)g + 2(7)f + c = 0$.

36 + 49 + 12g + 14f + c = 0.

12g + 14f + c = −85. … (2)

Since the centre is (−g, −f), substituting into 2y − x = 3:

−2f + g = 3.

g = 3 + 2f.

Substitute into equation (1):

4(3 + 2f) + 6f + c = −13.

12 + 8f + 6f + c = −13.

14f + c = −25. … (3)

Substitute into equation (2):

12(3 + 2f) + 14f + c = −85.

36 + 24f + 14f + c = −85.

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$38f + c = -121$. … (4)

Subtract equation (3) from equation (4):

$24f = -96$.

$f = -4$.

Then:

$g = 3 + 2(-4)$

$= 3 - 8$

$= -5$.

Substituting $f = -4$ into equation (3):

$14(-4) + c = -25$.

$-56 + c = -25$.

$c = 31$.

Therefore:

$x^2 + y^2 + 2(-5)x + 2(-4)y + 31 = 0$.

Equation of the circle: $x^2 + y^2 - 10x - 8y + 31 = 0$.

11(a)

Evaluate:

$[\log_2 8 + \log_2 16 - 4\log_2 2]/\log_4 16$.

Using $4\log_2 2 = \log_2 2^4$:

$= [\log_2 8 + \log_2 16 - \log_2 16]/\log_4 16$

$= \log_2[(8 \times 16)/16]/\log_4 16$

$= \log_2 8/\log_4 16$

$= \log_2 2^3/\log_4 4^2$

$= (3\log_2 2)/(2\log_4 4)$

$= (3 \times 1)/(2 \times 1)$

$= 3/2$.

Answer: 1½.

11(b)

For the arithmetic progression:

$T_1 = a$.

$T_3 = a + 2d$.

$T_7 = a + 6d$.

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If these form consecutive terms of a geometric progression:

$(a + 2d)/a = (a + 6d)/(a + 2d)$.

Therefore:

$a(a + 6d) = (a + 2d)^2$.

$a^2 + 6ad = a^2 + 4ad + 4d^2$.

$4d^2 - 2ad = 0$. … (*)

Also:

$T_1 + T_2 = 6$.

$a + a + d = 6$.

$2a + d = 6$.

$2a = 6 - d$.

Substituting into (*):

$4d^2 - (6 - d)d = 0$.

$4d^2 - 6d + d^2 = 0$.

$5d^2 - 6d = 0$.

$d(5d - 6) = 0$.

Therefore:

$d = 0$ or $d = 6/5$.

Then:

$a = (6 - d)/2$

$= (6 - 6/5)/2$

$= [(30 - 6)/5] \div 2$

$= (24/5) \times 1/2$

$= 12/5$.

First term = 12/5 = 2⅖.

Common difference = 6/5 = 1⅕.

PART II

12(a)

Farmers = 6.

Traders = 8.

Total = 14 people.

Required: a four-member executive.

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The probability of selecting at least two farmers is:

$P = [({}^6C_2)({}^8C_2) + ({}^6C_3)({}^8C_1) + ({}^6C_4)({}^8C_0)]/{}^{14}C_4$.

$= [(15 \times 28) + (20 \times 8) + (15 \times 1)]/1{,}001$.

$= (420 + 160 + 15)/1{,}001$.

$= 595/1{,}001$.

$= 85/143$.

Answer: 85/143.

12(b)

P(accident occurs in X) = 1/5.

P(accident does not occur in X) = 1 − 1/5 = 4/5.

P(accident occurs in Y) = 1/12.

P(accident does not occur in Y) = 1 − 1/12 = 11/12.

P(accident occurs in Z) = 1/6.

P(accident does not occur in Z) = 1 − 1/6 = 5/6.

(i) No accident in any of the factories

P = (4/5) × (11/12) × (5/6).

P = 11/18.

(ii) Accidents in all three factories

P = (1/5) × (1/12) × (1/6).

P = 1/360.

(iii) An accident in at least one factory

P = 1 − P(no accident in any factory)

= 1 − [(4/5) × (11/12) × (5/6)]

= 1 − 11/18.

P = 7/18.

13(a)

The ranks and differences in ranks are:

Candidate	Examiner 1	Examiner 2	Examiner 3	d_{12}	d_{12}^2	d_{13}	d_{13}^2	d_{23}	d_{23}^2
A	1	2	3	−1	1	−2	4	−1	1
B	3	1	2	2	4	1	1	−1	1

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Candidate	Examiner 1	Examiner 2	Examiner 3	d_{12}	d_{12}^2	d_{13}	d_{13}^2	d_{23}	d_{23}^2
C	6	3	1	3	9	5	25	2	4
D	2	9	6	−7	49	−4	16	3	9
E	10	7	9	3	9	1	1	−2	4
F	9	4	8	5	25	1	1	−4	16
G	7	8	7	−1	1	0	0	1	1
H	4	10	5	−6	36	−1	1	5	25
I	8	5	4	3	9	4	16	1	1
J	5	6	10	−1	1	−5	25	−4	16
Total					144		90		78

Thus:

$\Sigma d_{12}^2 = 144$.

$\Sigma d_{13}^2 = 90$.

$\Sigma d_{23}^2 = 78$.

n = 10.

Spearman's rank correlation coefficient:

$R_s = 1 - [6\Sigma d^2 / n(n^2 - 1)]$.

(i) Examiners 1 and 2

$R_s = 1 - [6 \times 144 / 10(100 - 1)]$

$= 1 - 864/990$

$= 126/990$

$= 7/55$

≈ 0.1273.

(ii) Examiners 1 and 3

$R_s = 1 - [6 \times 90 / 10(99)]$

$= 1 - 540/990$

$= 450/990$

$= 5/11$

≈ 0.45454545.

(iii) Examiners 2 and 3

$R_s = 1 - [6 \times 78 / 10(99)]$

$= 1 - 468/990$

$= 522/990$

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= 29/55

≈ 0.52727272.

13(b)

There is the greatest agreement between Examiners 2 and 3, whose rank correlation coefficient is approximately 0.5273, indicating a positive correlation.

PART III

14(a)

u = 10 m/s.

a = 0.2 m/s².

s = 100 m.

t = ?

Using:

$s = ut + \frac{1}{2}at^2$.

$100 = 10t + \frac{1}{2}(0.2)t^2$.

$100 = 10t + 0.1t^2$.

Multiplying through by 10:

$1,000 = 100t + t^2$.

$t^2 + 100t - 1,000 = 0$.

Using the quadratic formula:

$t = [-100 \pm \sqrt{(100^2 - 4 \times 1 \times -1,000)}]/2$

$= [-100 \pm \sqrt{(10,000 + 4,000)}]/2$

$= [-100 \pm \sqrt{14,000}]/2$

$\approx (-100 \pm 118.322)/2$.

Therefore:

$t \approx (-100 + 118.322)/2 = 9.161$.

Or:

$t \approx (-100 - 118.322)/2 = -109.161$.

Since time cannot be negative:

t = 9.16 seconds.

14(b)

$s = t^2 - 5t + 6$.

Velocity:

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$v = ds/dt = 2t - 5$.

When $t = 0$:

$v = 2(0) - 5$.

Initial velocity = −5 m/s.

At rest:

$ds/dt = 0$.

$2t - 5 = 0$.

$2t = 5$.

$t = 5/2 = 2.5$ seconds.

Substitute into the displacement equation:

$s = (5/2)^2 - 5(5/2) + 6$

$= 25/4 - 25/2 + 6$

$= (25 - 50 + 24)/4$

$= -1/4$ m.

Distance from the starting point = 0.25 m.

−0.25 m is the position relative to the coordinate origin at t = 2.5 s.

Since s(0) = 6 m, the displacement from the starting point is −6.25 m, and the distance travelled before coming to rest is 6.25 m.

15(a) Applying the cosine rule:

$3^2 = 1.8^2 + 2.4^2 - 2(1.8)(2.4)\cos\theta$.

$9 = 3.24 + 5.76 - 8.64 \cos\theta$.

$8.64 \cos\theta = 9 - 9 = 0$.

$\cos\theta = 0/8.64 = 0$.

$\theta = \cos^{-1}(0) = 90°$.

Therefore, $\angle XOY = 90°$.

Using the sine rule:

$1.8/\sin\alpha = 3/\sin 90°$.

$3 \sin\alpha = 1.8 \sin 90°$.

$\sin\alpha = 1.8/3 = 0.6$.

$\alpha = \sin^{-1}(0.6)$.

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α = 36.87°.

Also:

β + α + 90° = 180°.

β = 180° − (90° + 36.87°)

= 180° − 126.87°.

β = 53.13°.

15(b)

At equilibrium, the vector sum of the forces is zero:

(P sin 270°, P cos 270°)

(T sin 53.13°, T cos 53.13°)

(8 sin 180°, 8 cos 180°)

(T sin 323.13°, T cos 323.13°)

= (0, 0).

(−P, 0) + (0.8T, 0.6T) + (0, −8) + (−0.6T, 0.8T) = (0, 0).

Resolving horizontally:

−P + 0.8T − 0.6T = 0.

−P + 0.2T = 0.

P = 0.2T. … (i)

Resolving vertically:

0.8T + 0.6T − 8 = 0.

1.4T = 8.

T = 8/1.4.

T = 5.714 N.

Substituting into equation (i):

P = 0.2(5.714)

= 1.1428.

P = 1.143 N.

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WASSCE JUNE 2021 — FURTHER MATHEMATICS 2 (ESSAY)

SECTION A

Question 1

$f(x) = 2x^3+Px^2+Qx-5$

If $x-1=0 \Rightarrow x=1$; substituting in original polynomial

$f(1) = 2(1^3)+P(1^2)+Q(1)-5 = 0$

$\Rightarrow 2+P+Q-5 = 0$

$\Rightarrow P+Q = 3 \ldots (1)$

$x+2=0 \Rightarrow x=-2$; substituting in the polynomial

$f(-2) = 2(-2^3)+P(-2^2)+Q(-2)-5 = 27$

$\Rightarrow 2(-8)+P(4)+Q(-2)-5 = 27$

$-16+4P-2Q-5 = 27$

$4P-2Q-21 = 27$

$4P-2Q = 48$; dividing through by 2

$2P-Q = 24 \ldots (2)$

Adding equations (1) and (2):

$3P = 27$

$\therefore P = 27/3 = 9$

Put P=9 in equation (1):

$9+Q = 3$

$\Rightarrow Q = 3-9 = -6$

Hence, P=9, Q=-6

Question 2

$\int_1^9 x(2x-3)/\sqrt{x}\, dx$

Let $u = \sqrt{x}$, such that $u^2 = x$

$\Rightarrow 2u\, du = dx$

When x=1: $u=\sqrt{1}=1$

When x=9: $u=\sqrt{9}=3$; substituting

$\int_1^3 u^2(2u^2-3)/u\ (2u\, du)$

$= \int_1^3 (2u^4-3u^2)(2du)$

$= \int_1^3 (4u^4-6u^2)\, du$

$= [4u^5/5 - 6u^3/3]_1^3$

$= (4u^5/5 - 6u^3/3)$ from 1 to 3

$= [4/5(3^5) - 6/3(3^3)] - [4/5(1^5) - 6/3(1^3)]$

$= [(4/5)(243) - 2(27)] - [4/5 - 2]$

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= [(4/5×243) - 54] - (4/5 - 2/1)

= (972/5 - 54) - (-1⅕)

= (972-270)/5 + 1⅕

= 702/5 + 6/5

= (702+6)/5

= 708/5 = 141⅗ or 141.6

Question 3

$(P+½\sqrt{3})(1-\sqrt{3})^2 = 3-\sqrt{3}$

$(P+½\sqrt{3})(1-\sqrt{3}-\sqrt{3}+3) = 3-\sqrt{3}$

$(P+½\sqrt{3})(1-2\sqrt{3}+3) = 3-\sqrt{3}$

$(P+½\sqrt{3})(4-2\sqrt{3}) = 3-\sqrt{3}$

Expanding:

$4P-2P\sqrt{3}+2\sqrt{3}-3 = 3-\sqrt{3}$

$\Rightarrow 4P-2P\sqrt{3} = -2\sqrt{3}-\sqrt{3}+3+3$

$4P-2P\sqrt{3} = -3\sqrt{3}+6$

$2P(2-\sqrt{3}) = 6-3\sqrt{3}$

$2P(2-\sqrt{3}) = 3(2-\sqrt{3})$

Dividing through by $2-\sqrt{3}$:

$\Rightarrow 2P = 3$

$\therefore P = 3/2 = 1½$

Question 4

${}^{5y}C_2 = 190$

$\Rightarrow (5y)!/[2!(5y-2)!] = 190$

$\Rightarrow (5y)!/(5y-2)! = 2(190)$

$(5y)!/(5y-2)! = 380$

$\Rightarrow (5y)(5y-1)!/(5y-1)! = 380$

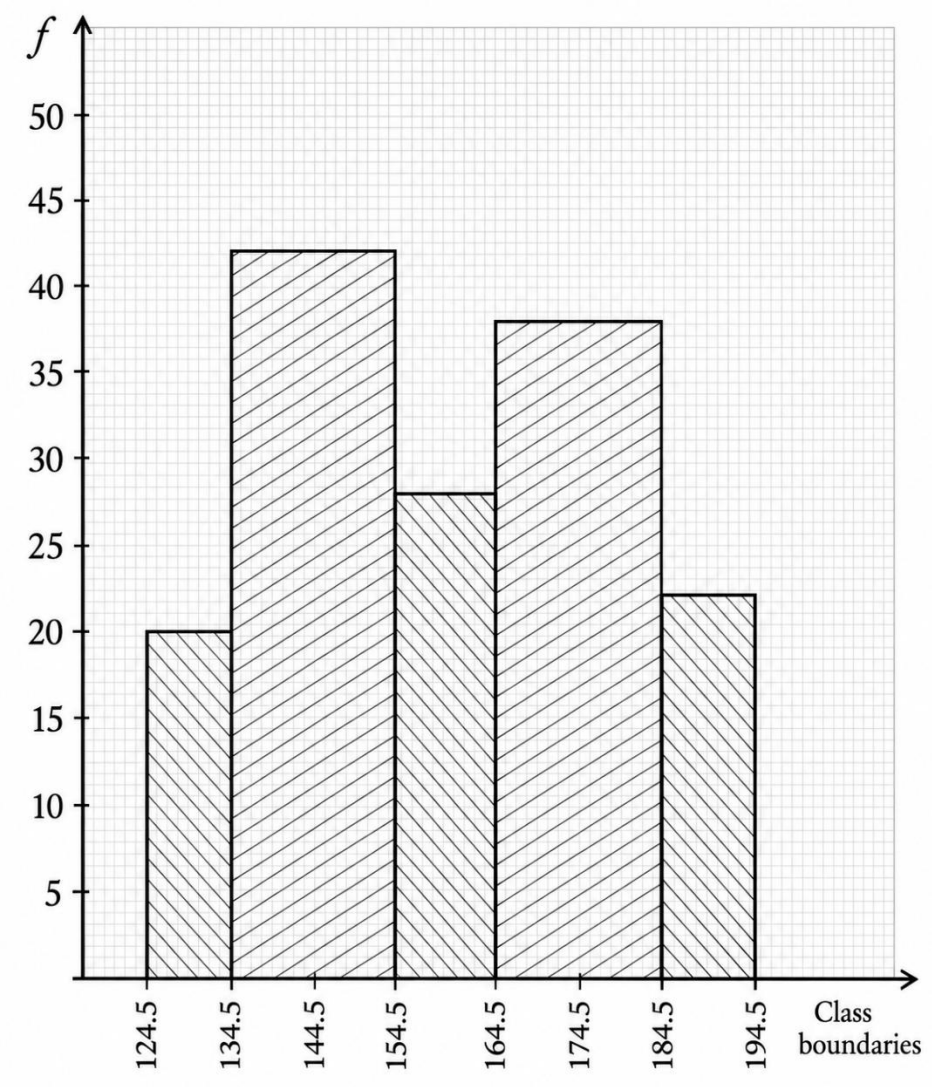
$\Rightarrow 5y = 380$

$\therefore y = 380/5 = 76$

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Question 5

(a)



[Histogram]

Monthly Income (₦'000)	f	Class boundary	Class Width	Frequency density
135-139	20	134.5-139.5	5	4
140-149	42	139.5-149.5	10	4.2
150-154	28	149.5-154.5	5	5.6
155-164	38	154.5-164.5	10	3.8
165-169	22	164.5-169.5	5	4.4

(b) From the histogram: Mode = 152.5

Question 6

Total number of mangoes = 24

Bad mangoes = 6

⇒ Good mangoes = 18

Pr(a bad mango) = 6/24 = ¼

⇒ Pr(a good mango) = 1-¼ = ¾

Let Pr(Bad Mango) = P = ¼

Let Pr(Good Mango) = Q = ¾

$= (P+Q)^6$

Coefficients = 1,6,15,20,15,6,1

$= P^6+6P^5Q+15P^4Q^2+20P^3Q^3+15P^2Q^4+6PQ^5+Q^6$

$= (¼)^6+6(¼)^5(¾)+15(¼)^4(¾)^2+20(¼)^3(¾)^3+15(¼)^2(¾)^4+6(¼)(¾)^5+(¾)^6$

Pr(Not More than 3 bad) = Pr(P≤3 bad)

$Pr(P\leq3) = 20P^3Q^3+15P^2Q^4+6PQ^5+Q^6$

$= 20(¼)^3(¾)^3+15(¼)^2(¾)^4+6(¼)(¾)^5+(¾)^6$

$= 20(1/64)(27/64)+15(1/16)(81/256)+6(¼)(243/1024)+729/4096$

$= 20\times27/(64\times64) + 15\times81/(16\times256) + 6\times243/(4\times1024) + 729/4096$

$= 540/4096 + 1215/4096 + 1458/4096 + 729/4096$

$= (540+1215+1458+729)/4096$

$= 3942/4096$

$= 0.9624023$

$= 0.9624$

Question 7

u = 45m/s; S = ?

v = 5m/s

$a = -10m/s^2$

Distance, using $v^2=u^2+2aS$

$\Rightarrow 5^2 = (45)^2+2(-10)S$

$25 = 2025-20S$

$20S = 2025-25$

$20S = 2000$

$\therefore S = 2000/20 = 100$

$\Rightarrow$ Distance covered = 100m

Question 8

X = (-4, 3) and Y = (-9, -15)

X.Y = (-4,3)·(-9,-15) = (36,-45) = 36-45 = -9

But $|X| = \sqrt{[(-4)^2+(3)^2]} = \sqrt{(16+9)} = \sqrt{25} = 5$

$|Y| = \sqrt{[(-9)^2+(-15)^2]} = \sqrt{(81+225)} = \sqrt{306}$

From Direction Cosine (Dot Product):

$\cos\theta = -9/(5\sqrt{306}) = -9/5(17.4929) = -9/87.464$

$\Rightarrow \cos\theta = -0.1029$

$\therefore \theta = \cos^{-1}(-0.1029) = 95.91$

$\Rightarrow \theta \approx 96°$

SECTION B

Question 9(a) (PART I)

U_n = 15km = 15000m (15×1000)

a=500; d=250

This forms an Arithmetic Progression

$U_n = a+(n-1)d$

$\Rightarrow 15000 = 500+(n-1)250$

$\Rightarrow 15000 = 500+250n-250$

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15000+250-500 = 250n

⇒ 250n = 14750

∴ n = 14750/250 = 59

Including the 1st day: n-1 = 59

∴ n = 59+1

No of days = 60 days

Total distance = sum of the sequence

S_{59} = n/2{2a+(n-1)d}

= 59/2{2(500)+(59-1)250}

= 59/2(1000+58×250)

= 59/2(1000+14,500)

= 59/2(15,500)

= 59(7,750) = 457,250m

Converting to kilometres:

⇒ Total Distance = 457,250/1000 = 457.25km

Question 9(b)

T_2 = ar = -3 ⇒ a = -3/r ...(1)

S_∞ = a/(1-r) = 25/2 ⇒ a = 25(1-r)/2 ...(2)

Comparing equations (1) and (2):

⇒ -3/r = 25(1-r)/2

25(1-r)r = -6

(25-25r)r = -6

⇒ 25r-25r^2 = -6

25r^2-25r-6 = 0

a=25, b=-25, c=-6

Using r = [-b±√(b^2-4ac)]/2a

⇒ r = [-(-25)±√((-25)^2-4×25×-6)]/(2×25)

= [25±√(625+600)]/50

= [25±√1225]/50

= (25±35)/50

⇒ r = (25+35)/50 or (25-35)/50

= 60/50 or -10/50

∴ r = 6/5 or -⅕

Question 10

(a) Matrices P and Q:

P =

-3	**6**
4	1

Q =

2	**-3**
-4	-6

(b) Q(-2,-3) = (2,-3; -4,-6)(-2,-3) = (-4+9, 8+18) = (5, 26)

∴ The image of (-2, -3) = (5, 26)

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(c) PQ:

-30	**-27**
4	-18

PQ: (x,y) ⇒ (-30x-27y, 4x-18y)

(d) (-30,-27; 4,-18)(1,4) = (-30-108, 4-72) = (-138,-68)

Hence, the image of the point (1,4) is (-138,-68)

(e) T = (cos225, -sin225; sin225, cos225) = (-√2/2, √2/2; -√2/2, -√2/2)

= (-√2, 2√2)(1+2, 1-2) = (3,-1)

∴ The image of P (P') is (3,-1)

Question 11(a)

Equation of the normal to the curve $y = (x^2-x+1)(x-2) = 0$

⇒ Either x-2=0 ⇒ x=2

or $x^2-x+1=0 \Rightarrow x^2-x+1=0$

Applying product rule:

dy/dx = u(dv/dx) + v(du/dx)

where $u=x^2-x+1$, $du/dx=2x-1$

v=x-2, dv/dx=1

Hence $dy/dx = (x^2-x+1)(1) + (x-2)(2x-1)$

$= x^2-x+1+2x^2-x-4x+2$

$\Rightarrow dy/dx = 3x^2-6x+3$

At $x=2$; $dy/dx = 3(2)^2-6(2)+3 = 12-12+3$

⇒ dy/dx = 3

Gradient of the normal = -1/(dy/dx) = -⅓

When x=2, y=0 ⇒ (x,y) = (2,0)

Hence, equation of the normal:

y = -⅓(x-2)

⇒ 3y = -x+2

or 3y+x-2 = 0

Question 11(b)

P(-1,2); Q(5,1); R(3,-4)

Midpoint of PR = ½(-1+3), ½(2-4)

= ½(2), ½(-2) = (1,-1)

Midpoint = (1,-1) = (x_1,y_1), Q(5,1) = (x_2,y_2)

Gradient, $M = (y_2-y_1)/(x_2-x_1) = (1-(-1))/(5-1) = 2/4 = ½$

Equation ⇒ $y = M(x-x_1)+y_1$

⇒ y = ½(x-5)+1

Hence, 2y = x-5+2

⇒ 2y = x-3

or 2y-x+3 = 0 — the equation

PART II

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Question 12(a)

Red = 5

Blue = 7

Green = 4

Total = 16

Pr(Same Colour) = Pr(RR)+Pr(BB)+Pr(GG)

= (5/16×4/15)+(7/16×6/15)+(4/16×3/15)

= 20/240 + 42/240 + 12/240

⇒ Pr(Same Colour) = (20+42+12)/240 = 74/240 = 37/120

Height (cm)	Frequency (f)	Class boundary (c.b)	Cumulative frequency (c.f)
100-109	27	99.5-109.5	27
110-119	58	109.5-119.5	85
120-129	130	119.5-129.5	215
130-139	105	129.5-139.5	320
140-149	50	139.5-149.5	370
150-159	25	149.5-159.5	395
160-169	5	159.5-169.5	400

Question 12(b)

Pr(Different Colour) = (5/16×7/15)+(5/16×4/15)+(7/16×4/15)

= 35/120+20/120+28/120

= (35+20+28)/120 = 83/120

Question 12(c)

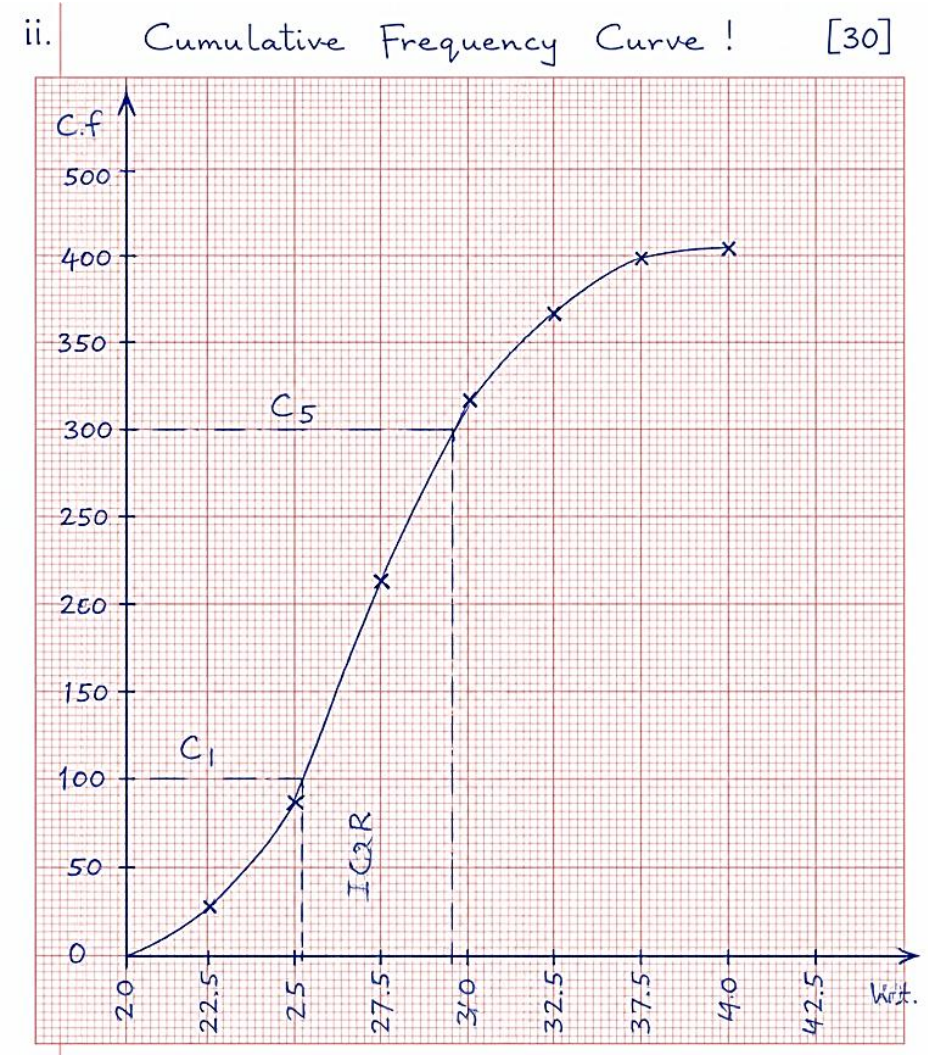
Pr(at least one red) = (5/16×7/15)+(5/16×4/15)+(5/16×11/15) [using $^{16}C_2$ combinations]

= (35+20+10)/120 = 65/120 = 13/24

Question 13(a)(i)

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Question 13(a)(ii)



[Cumulative Frequency Curve]

(b)(i) Median height = (N+1)/2 th = (400+1)/2 th = 200.5th

From the Curve = 128.5

Question 13(b)(ii)

Interquartile Range = Q_3-Q_1

Q_3 = ¾N = ¾×400 = 300th position

Q_1 = ¼N = ¼×400 = 100th position

Q_3-Q_1 = 137.5-120.5

= 17.0

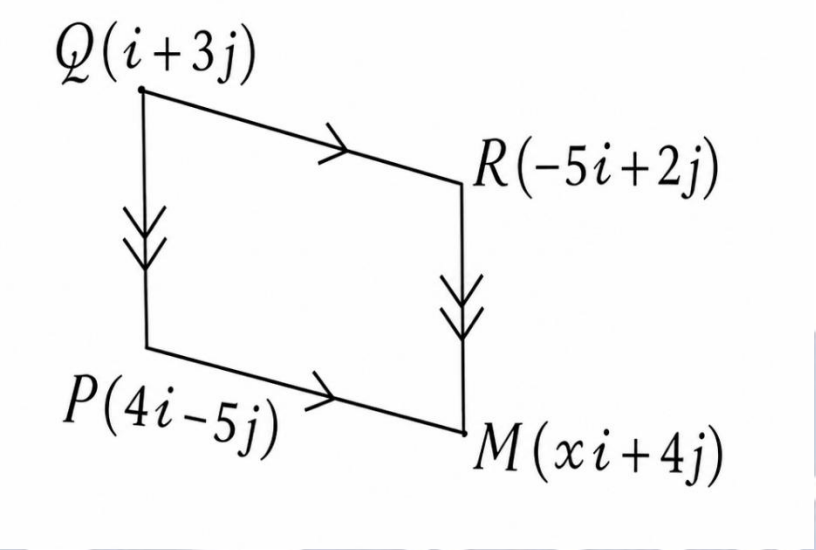
Question 13(b)(iii)

Percentage of students whose height are at most 130cm (≤130cm)

= (27+58+130)/400 × 100/1

= 215/400 × 100 = 53.75%

Question 14(a)



Q(i+3j), R(-5i+2j), P(4i-5j), M(xi+yj)

PQ = (1,3)-(4,-5) = (-3,8)

MR = (-5,2)-(x,y) = (-5-x, 2-y)

⇒ (-3,8) = (-5-x, 2-y)

Hence, -5-x=-3 ⇒ x=3-5

and 2-y=8 ⇒ y=2-8

⇒ x=-2 and y=-6

∴ Coordinate of M(-2,-6)

Question 14(b)

$\vec{PM}$ = $\vec{OM}$-$\vec{OP}$ = (-2,-6)-(4,-5) = (-6,-1)

$\vec{PQ}$ = $\vec{OQ}$-$\vec{OP}$ = (1,3)-(4,-5) = (-3,8)

$|\vec{PM}| = \sqrt{[(-6)^2+(-1)^2]} = \sqrt{(36+1)} = \sqrt{37}$

$|\vec{PQ}| = \sqrt{[(-3)^2+(8)^2]} = \sqrt{(9+64)} = \sqrt{73}$

$\cos\theta = (\vec{PM}\cdot\vec{PQ})/(|\vec{PM}||\vec{PQ}|) = [(-6i-j)\cdot(3i+8j)]/(\sqrt{37}\cdot\sqrt{73}) = (18-8)/\sqrt{(37\times73)}$

⇒ $\cos\theta = 10/\sqrt{2701} = 10/51.97115 = 0.19244$

⇒ $\cos\theta = 0.192414$

∴ $\theta = \cos^{-1}(0.1924) = 78.918798 \approx 78.92°$

PART III

Question 15(a)

From the top of a cliff:

u=0; S=50; g=10m/s²; V=30m/s

Using $S = ut+\frac{1}{2}gt^2$

⇒ $50 = 0(t)+\frac{1}{2}\times10t^2$

$50 = 5t^2$

⇒ $t^2 = 50/5 = 10$

∴ $t = \sqrt{10} = 3.162$

Horizontal distance R = speed×time

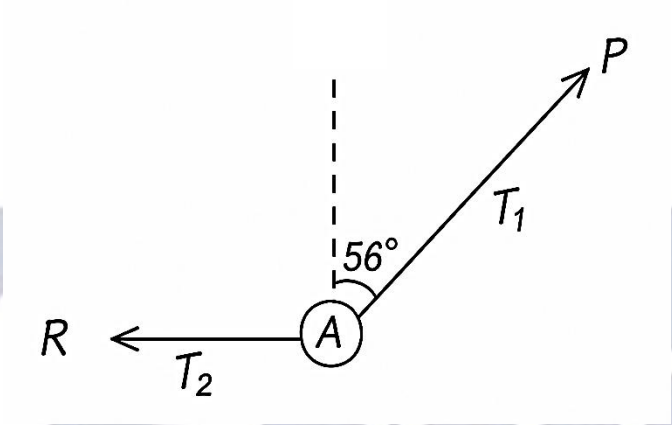
⇒ R = 30×3.1623

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= 94.8683298

∴ R ≈ 94.87m

Question 15(b)



Mass = 2kg

θ = 56°

g = 10m/s²

Weight of the body, W = 2×10 = 20N

Resolving the forces horizontally:

$T_2 = T_1 Sin\Theta = T_1 Sin56°$

$\Rightarrow T_2 = T_1 Sin56°$ ...(1)

Resolving the forces vertically:

$T_1 Cos56° = W$

$\Rightarrow T_1 Cos56° = W = 20$ ...(2)

Hence, $T_1 = 20/Cos56° = 20/0.559192$

$= 20/0.5592 = 35.7654$

$\Rightarrow T_1 = 35.77N$

But from equation (1),

$T_2 = T_1 Sin56°$

$= 35.77 \times 0.829038$

$= 29.65467$

$\therefore T_2 \approx 29.65N$

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WASSCE JUNE 2022 — FURTHER MATHEMATICS 2 (ESSAY)

SECTION A

SECTION A

QUESTION 1

(a) Table for the operation * ($a * b = a^2 + 2ab - b^2$):

*	−2	−1	1	2
−2	8	7	−1	−8
−1	1	2	−2	−7
1	−7	−2	2	1
2	−8	−1	7	8

(b) $p * 2 = p^2 + 2p(2) - 2^2$, and $(-2 * p) * 2 = -7$.

Let $X = -2 * p$, so $X * 2 = -7$.

$X * 2 = X^2 + 4X - 4 = -7$

$X^2 + 4X + 3 = 0$

$(X + 1)(X + 3) = 0$

$X = -1$ or $X = -3$

Recall $X = -2 * p$, so $-2 * p = -1$.

∴ $P = 2 - 1 = 1$ (as written on the page)

From the table:

$-2 * p = 4 - 4p - p^2 = -1$

$p^2 + 4p - 5 = 0$

$(p - 1)(p + 5) = 0$

$p = -5$ or 1

Hence $p = 1$

QUESTION 2

$2^{(2y+1)} - 5(2^y) + 2 = 0$

$2^1(2^{(2y)}) - 5(2^y) + 2 = 0$

Let $2^y = P$:

$2P^2 - 5P + 2 = 0$

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Factorising:

$(P - 2)(2P - 1) = 0$

Either $P - 2 = 0$, so $P = 2$, or $2P - 1 = 0$, so $P = ½$.

But $P = 2^y$.

At $P = 2$: $2^y = 2$, so $y = 1$.

At $P = ½ = 2^{-1}$: $2^y = 2^{-1}$, so $y = -1$.

$\therefore y = 1$ or -1

QUESTION 3

$(g \circ f)(x) = 1/(x^2 + 4)$ (the first line is faint, so I read it as this)

$y = 1/(x^2 + 4)$

$1 = yx^2 + 4y$

$1 - 4y = yx^2$

$x^2 = (1 - 4y)/y$

$x = \pm\sqrt{((1 - 4y)/y)}$

$\Rightarrow (g \circ f)^{-1}(x) = \pm\sqrt{((1 - 4x)/x)}$, $x \neq 0$

If $x < 0$, $1/x - 4 < 0$, which is undefined.

If $x > 0$, $1/x - 4 \geq 0$.

Therefore, the domain = $0 < x \leq ¼$

QUESTION 4

$3\cos 2x - \sin x = 0$

But $\cos 2x = 1 - 2\sin^2 x$

$3(1 - 2\sin^2 x) - \sin x = 0$

$3 - 6\sin^2 x - \sin x = 0$

$6\sin^2 x + \sin x - 3 = 0$

Let $y = \sin x$:

$6y^2 + y - 3 = 0$

By the formula method ($a = 6$, $b = 1$, $c = -3$):

$y = (-b \pm \sqrt{(b^2 - 4ac)})/2a$

$= (-1 \pm \sqrt{(1^2 - 4(6)(-3))})/(2(6))$

$= (-1 \pm \sqrt{(1 + 72)})/12$

$= (-1 \pm \sqrt{73})/12$

$= (-1 \pm 8.544)/12$

Hence $y = (-1 + 8.544)/12 = 7.544/12 = 0.628667$

or $y = (-1 - 8.544)/12 = -9.544/12 = -0.795333$

When $y = 0.6287$:

$\sin x = 0.6287$

$x = \sin^{-1}(0.6287) = 38.9543$

$x = 38.95°$ or $141.05°$

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When y = −0.7953:

sin x = −0.7953

$x = \sin^{-1}(-0.7953)$

x = 232.68° or 307.32°

QUESTION 5

Let p be the probability that Abiola will be late to the office, and q the probability that she will not be late.

p = 2/5 and q = 1 − 2/5 = 3/5

n = 6; r = 3

$^nC_r\ p^r\ q^{n-r}$

(a) For 3 days lateness:

$P(r = 3) = {}^6C_3\ (2/5)^3(3/5)^3$

= 20 × 8/125 × 27/125

= 864/3125

= 0.2765 (4 s.f.)

(b) $P(r = 0) = {}^6C_0\ (2/5)^0(3/5)^6$

= 1 × 1 × 729/15625

= 0.046656

= 0.0466 (4 s.f., as written)

(c) $P(r = 6) = {}^6C_6\ (2/5)^6(3/5)^0$

= 1 × 64/15625 × 1

= 64/15625

= 0.004096 (4 s.f.)

QUESTION 6

Rx	Ry	d = Rx − Ry	d^2
7	5	2	4
5	4	1	1
4	3	1	1
8	8	0	0
3	7	−4	16
2	2	0	0
1	1	0	0
6	6	0	0
			$\Sigma d^2 = 22$

Applying $r = 1 - 6\Sigma d^2/(N(N^2 - 1))$:

$= 1 - 6(22)/(8(8^2 - 1))$

$= 1 - 132/(8(63))$

$= 1 - 132/504$

$= (504 - 132)/504$

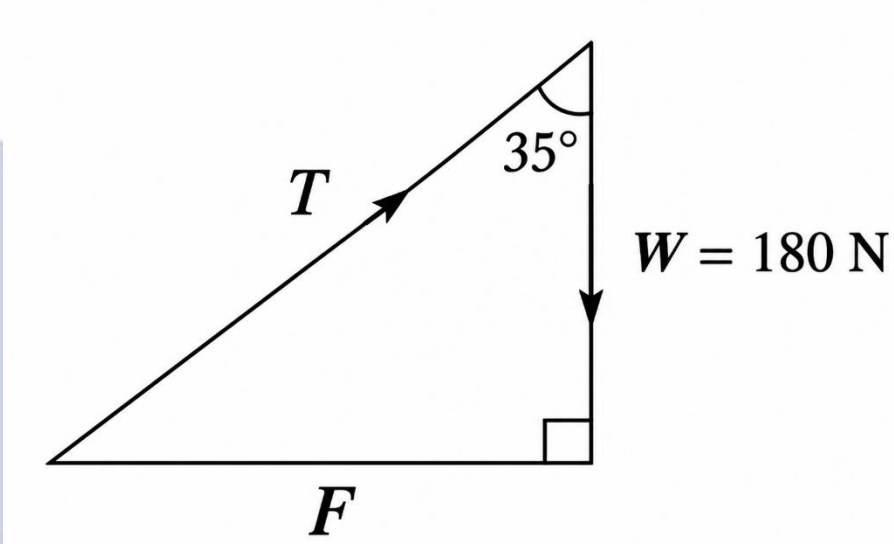
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= 372/504

= 0.738095

r = 0.7381

QUESTION 7



(a triangle with a 35° angle, tension T, force F and weight W = 180 N)

From trigonometry:

F/180 = tan 35°

F = 180 tan 35°

= 180 × 0.7002075

= 126.03735

F = 126.04 N

Again, 180/T = cos 35°

T cos 35° = 180

T = 180/cos 35°

= 180/0.819152

= 219.73997

T = 219.74 N

QUESTION 8

P = (8 N, 030°) = (8 sin 30°, 8 cos 30°) = (8 × 0.5, 8 × 0.866) = (4, 6.928)

Q = (9 N, 150°) = (9 sin 150°, 9 cos 150°) = (9 × 0.5, 9 × −0.866) = (4.5, −7.7942)

Now P − Q = (4, 6.928) − (4.5, −7.7942) = (−0.5, 14.7222)

Unit vector of (P − Q), where the unit vector is given by (xi + yj)/|xi + yj|:

= (−0.5i + 14.7222j)/√((0.5)² + (14.7222)²)

= (−0.5i + 14.7222j)/√(0.25 + 216.7431)

= (−0.5i + 14.7222j)/14.731

SECTION B

QUESTION 9

$\binom{n}{4}, \binom{n}{5}, \binom{n}{6}$, i.e. ${}^nC_4, {}^nC_5, {}^nC_6$, are in arithmetic progression.

The common difference $d = \binom{n}{5} - \binom{n}{4} = \binom{n}{6} - \binom{n}{5}$

Collecting like terms:

$\binom{n}{5} + \binom{n}{5} = \binom{n}{6} + \binom{n}{4}$

$2\binom{n}{5} = \binom{n}{6} + \binom{n}{4}$

${}^nC_x = n!/(x!(n - x)!)$

$2n!/(5!(n - 5)!) = n!/(6!(n - 6)!) + n!/(4!(n - 4)!)$

Dividing through by n!:

$2/(5!(n - 5)!) = 1/(6!(n - 6)!) + 1/(4!(n - 4)!)$

$2/(5!(n - 5)!) = 1/(4!(n - 6)!) \times (1/30 + 1/((n - 4)(n - 5)))$

$= 1/(4!(n - 6)!) \times ((n - 4)(n - 5) + 30)/(30(n - 4)(n - 5))$

Simplifying:

$1 = (5/2) \times (n^2 - 9n + 20 + 30)/(30(n - 4))$

$(2/5)(30(n - 4)) = n^2 - 9n + 50$

$12n - 48 = n^2 - 9n + 50$

$n^2 - 21n + 98 = 0$

Solving by factorisation:

$(n - 14)(n - 7) = 0$

Either $n - 14 = 0$, so $n = 14$, or $n - 7 = 0$, so $n = 7$.

$\therefore n = 14$ or 7

(b) When $n = 14$: $d = \binom{14}{5} - \binom{14}{4} = 2002 - 1001 = 1001$

When $n = 7$: $d = \binom{7}{5} - \binom{7}{4} = 21 - 35 = -14$

QUESTION 10

(a) Volume of rectangular box = L × B × H

$72 = 3x(2x)y$

$72 = 6x^2y$

$72/6 = x^2y$

$x^2y = 12$

$y = 12/x^2$

(b)(i) S.A. = 2(Lb + bh + Lh). Let the total surface area be A.

$A = 2(6x^2 + 2xy + 3xy)$

$= 2(6x^2 + 5xy)$

$= 12x^2 + 10xy$

But $y = 12/x^2$, substituting:

$A = 12x^2 + 10x(12/x^2)$

$= 12x^2 + 120/x$

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Now $dA/dx = 24x - 120/x^2$

(b)(ii) For stationary value, $dA/dx = 0$:

$24x - 120/x^2 = 0$

$24x^3 = 120$

$x^3 = 120/24$

$x = \sqrt[3]{(120/24)} = \sqrt[3]{5}$

$x = 1.71$

QUESTION 11

(a) $(1 + 2x)^7$

Coefficients: 1, 7, 21, 35, 35, 21, 7, 1

$= 1(1^7)(2x)^0 + 7(1^6)(2x)^1 + 21(1^5)(2x)^2 + 35(1^4)(2x)^3 + 35(1^3)(2x)^4 +$

$21(1^2)(2x)^5 + 7(1^1)(2x)^6 + 1(1^0)(2x)^7$

$= 1 + 7(2x) + 21(4x^2) + 35(8x^3) + 35(16x^4) + 21(32x^5) + 7(64x^6) + 128x^7$

$= 1 + 14x + 84x^2 + 280x^3 + 560x^4 + 672x^5 + 448x^6 + 128x^7$

$(1 - 2x)^7 = 1(1^7)(-2x)^0 + 7(1^6)(-2x)^1 + 21(1^5)(-2x)^2 + 35(1^4)(-2x)^3 +$

$35(1^3)(-2x)^4 + 21(1^2)(-2x)^5 + 7(1^1)(-2x)^6 + 1(1^0)(-2x)^7$

$= 1 - 14x + 84x^2 - 280x^3 + 560x^4 - 672x^5 + 448x^6 - 128x^7$

From the results obtained above:

$(1 + 2x)^7 - (1 - 2x)^7$

$= (1 - 1) + (14x + 14x) + (84x^2 - 84x^2) + (280x^3 + 280x^3) + (560x^4 - 560x^4)$

$+ (672x^5 + 672x^5) + (448x^6 - 448x^6) + (128x^7 + 128x^7)$

$= 28x + 560x^3 + 1344x^5 + 256x^7$

(b) Comparing $(1 + 2x)^7 - (1 - 2x)^7$ and $(1.2)^7 - (0.8)^7$:

Either $1 + 2x = 1.2$, so $2x = 1.2 - 1 = 0.2$,

or $1 - 2x = 0.8$, so $2x = 1 - 0.8 = 0.2$.

In each case, $2x = 0.2$, so $x = 0.2/2 = 0.1$

$(1.2)^7 - (0.8)^7 = 28x + 560x^3 + 1344x^5 + 256x^7$

Substituting for $x = 0.1$:

$= 28(0.1) + 560(0.1)^3 + 1344(0.1)^5 + 256(0.1)^7$

$= 2.8 + 0.56 + 0.01344 + 0.0000256$

$= 3.3744256$ (as written)

$= 3.374$ (3 decimal places, as written)

PART II

QUESTION 12

Let the number of oranges be x, the number of apples be x + 1, and the number of avocado pears be x + 2.

$x + (x + 1) + (x + 2) = 12$

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3x + 3 = 12

3x = 12 − 3 = 9

x = 3

Oranges = 3, Apples = 3 + 1 = 4, Avocado pears = 3 + 2 = 5

(a) Pr(1st orange, 2nd avocado pear) = 3/12 × 5/11 = 5/44

(b) Pr(both are the same fruit) = Pr(both oranges) or Pr(both apples) or Pr(both avocado)

= (3/12 × 2/11) + (4/12 × 3/11) + (5/12 × 4/11)

= 1/22 + 1/11 + 5/33

= (3 + 6 + 10)/66

= 19/66

(c) Pr(at least one apple) = Pr(2 apples) or Pr(1 apple and 1 orange) or Pr(1 apple and 1 avocado)

= (4/12 × 3/11) + (4/12 × 3/11) + (4/12 × 5/11)

= 1/11 + 1/11 + 5/33

= (3 + 3 + 5)/33

= 11/33 (as written)

QUESTION 13

(a) Data:

x	**14**	**16**	**17**	**18**	**22**	**24**	**27**	**28**	**31**	**33**
y	22	19	15	13	10	12	3	5	3	2

(b)(i) $\bar{x}$ = (14 + 16 + 17 + 18 + 22 + 24 + 27 + 28 + 31 + 33)/10

= 230/10

$\bar{x}$ = 23

(b)(ii) $\bar{y}$ = (22 + 19 + 15 + 13 + 10 + 12 + 3 + 5 + 3 + 2)/10

= 104/10

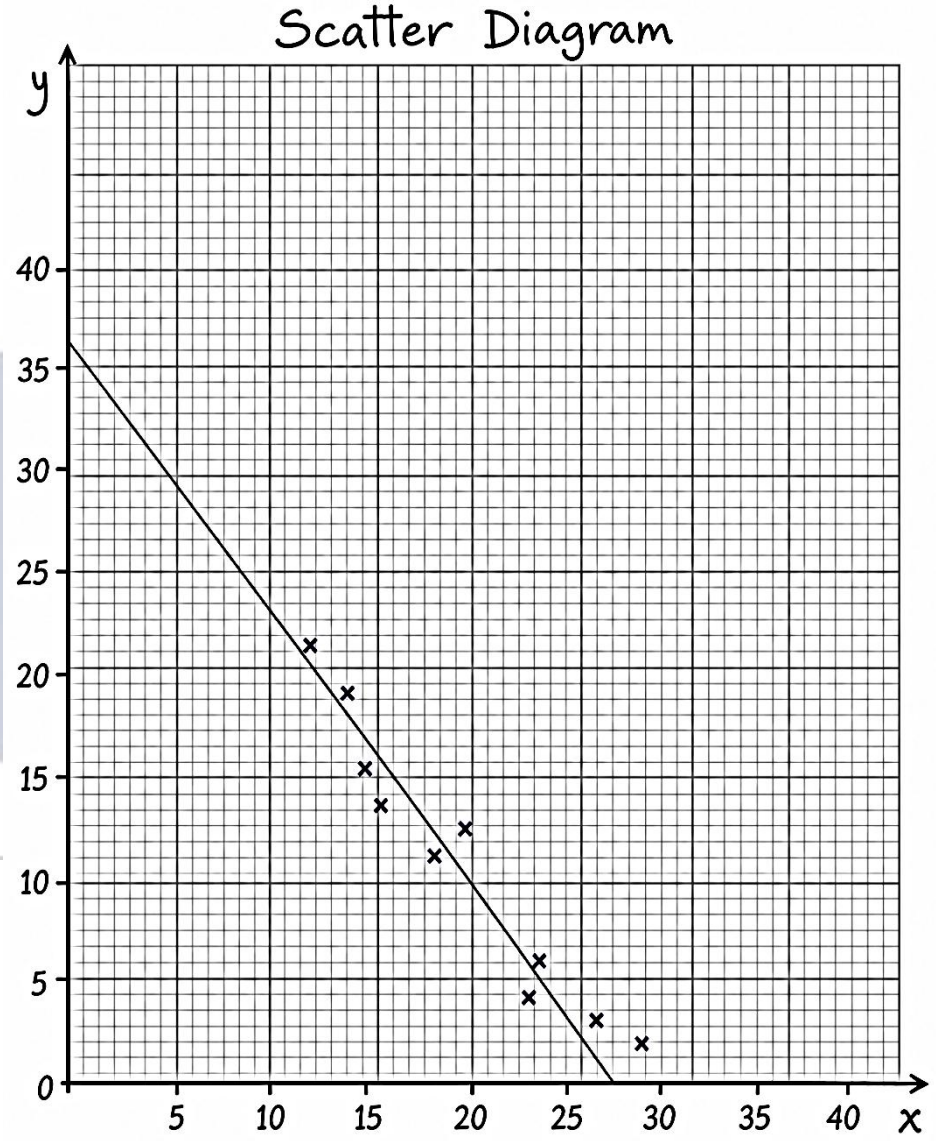
$\bar{y}$ = 10.4

$\bar{x}_1$ = 87/5 = 17.4

$\bar{y}_1$ = 79/5 = 15.8

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(c) Scatter diagram of the data, with the line of best fit drawn.



(d)(i) Gradient = $(\bar{y}_1 - \bar{y})/(\bar{x}_1 - \bar{x})$

= (15.8 − 10.4)/(17.4 − 23)

= 5.4/(−5.6)

= 2.7/(−2.8)

(d)(ii) Line of best fit: $y - \bar{y} = m(x - \bar{x})$

y − 10.4 = (2.7/−2.8)(x − 23)

−2.8(y − 10.4) = 2.7(x − 23)

−2.8y + 29.12 = 2.7x − 62.1

−2.8y = 2.7x − 62.1 − 29.12

−2.8y = 2.7x − 91.22

2.8y = −2.7x + 91.22

At x = 20:

2.8y = −2.7(20) + 91.22

2.8y = −54.0 + 91.22

2.8y = 37.22

y = 37.22/2.8

y = 13.29

PART III

QUESTION 14

(a)(i) $a = 10t - 4t^2$, but $a = dv/dt$

$\int dv = \int (10t - 4t^2)\, dt$

$v = 10t^2/2 - 4t^3/3 + K$

$= 5t^2 - (4/3)t^3 + K$

At rest, $t = 0$ and $v = 0$:

$5(0)^2 - (4/3)(0)^3 + K = 0$

$K = 0$

Hence $v = 5t^2 - (4/3)t^3$, the velocity at time t.

(a)(ii) $a = (v_4 - v_3)/(t_4 - t_3)$

$v_4 = 5(4)^2 - (4/3)(4)^3$

$= 5(16) - (4/3)(64)$

$= 80 - 256/3$

$= 80 - 85.33$

$= -5.33$ m/s

$v_3 = 5(3)^2 - (4/3)(3)^3$

$= 5(9) - (4/3)(27)$

$= 45 - 36$

$= 9$ m/s

Average acceleration $= (-5.33 - 9)/(4 - 3)$

$= -14.33/1$

$= -14.33$ m/s²

(b) $R - mg = ma$ ……(1)

$R - 120 \times 10 = 0$

$R - 1200 = 0$

$R = 1200$ N

When $a = 3$ m/s², substituting from equation (1):

$R - 120 \times 10 = 120 \times 3$

$R - 1200 = 360$

$R = 360 + 1200$

$R = 1560$ N

QUESTION 15

(a) $P = 6i + 8j$, $Q = 8i - 6j$

$|P| = \sqrt{(6^2 + 8^2)} = \sqrt{(36 + 64)} = \sqrt{100} = 10$

$|Q| = \sqrt{(8^2 + (-6)^2)} = \sqrt{(64 + 36)} = \sqrt{100} = 10$

Unit vector in the direction of $6i + 8j$

$= (1/10)(6i + 8j)$

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= (6/10)i + (8/10)j

= (3/5)i + (4/5)j

|OP| = 80:

OP = 80 × (1/10)(6i + 8j)

= 8(6i + 8j)

= 48i + 64j

|OQ| = 120:

OQ = 120 × (1/10)(8i − 6j)

= 12(8i − 6j)

= 96i − 72j

PQ = OQ − OP

= (96, −72) − (48, 64)

= (48, −136)

|PQ| = √(48² + (−136)²)

= √(2304 + 18496)

= √20800

|PQ| = 144.222

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WASSCE JUNE 2023 — FURTHER MATHEMATICS 2 (ESSAY)

SECTION A

SECTION A

QUESTION 1

${}^nC_x = n!/((n - x)!\, x!)$

${}^9C_x = 4({}^7C_{x-1})$

$9!/((9 - x)!\, x!) = 4 \times 7!/((7 - x + 1)!(x - 1)!)$

Cross multiplying:

$9!/(4 \times 7!) = (9 - x)!\, x!/((8 - x)!(x - 1)!)$

$(9 \times 8 \times 7!)/(4 \times 7!) = (9 - x)!\, x!/((8 - x)!(x - 1)!)$

$18 = (9 - x)(x)$

$18 = 9x - x^2$

$x^2 - 9x + 18 = 0$

$(x - 3)(x - 6) = 0$

Either $x - 3 = 0$ or $x - 6 = 0$

$x = 3$ or 6

QUESTION 2

Volume of cube, $V = s \times s \times s = s^3$

Rate of change, $dV/ds = 3s^2$

But $dV/dt = 7/2\ cm^3/s$

Recall $dV/dt = dV/ds \times ds/dt$

$7/2 = 3s^2\ ds/dt$

$ds/dt = 7/(6s^2)$

$= 7/(6 \times 6 \times 6)$

$= 7/216$ cm/s

QUESTION 3

(a) $f(x) = (5x - 6)/(4 - x);\ x \neq 4$

Let $y = (5x - 6)/(4 - x)$, then interchange x and y:

$x = (5y - 6)/(4 - y)$

$x(4 - y) = 5y - 6$

$4x - xy = 5y - 6$

$4x + 6 = 5y + xy$

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$4x + 6 = y(5 + x)$

$y = (4x + 6)/(5 + x)$

$f^{-1}(x) = (4x + 6)/(x + 5)$, with $x \neq 4$ as written on the page

(b) $(4x + 6)/(x + 5) = 5$

$5(x + 5) = 4x + 6$

$5x + 25 = 4x + 6$

$5x - 4x = 6 - 25$

$x = -19$

QUESTION 4

(a) The sum of an A.P. is given by $S_n = n/2(a + L)$, where $S_n = 286$, $a = -8$,

$L = 52$, $n = ?$

$286 = n/2(-8 + 52)$

$286 = n/2(44)$

$286 = 22n$

$n = 286/22 = 13$

(b) $T_n = a + (n - 1)d$

$52 = -8 + (13 - 1)d$

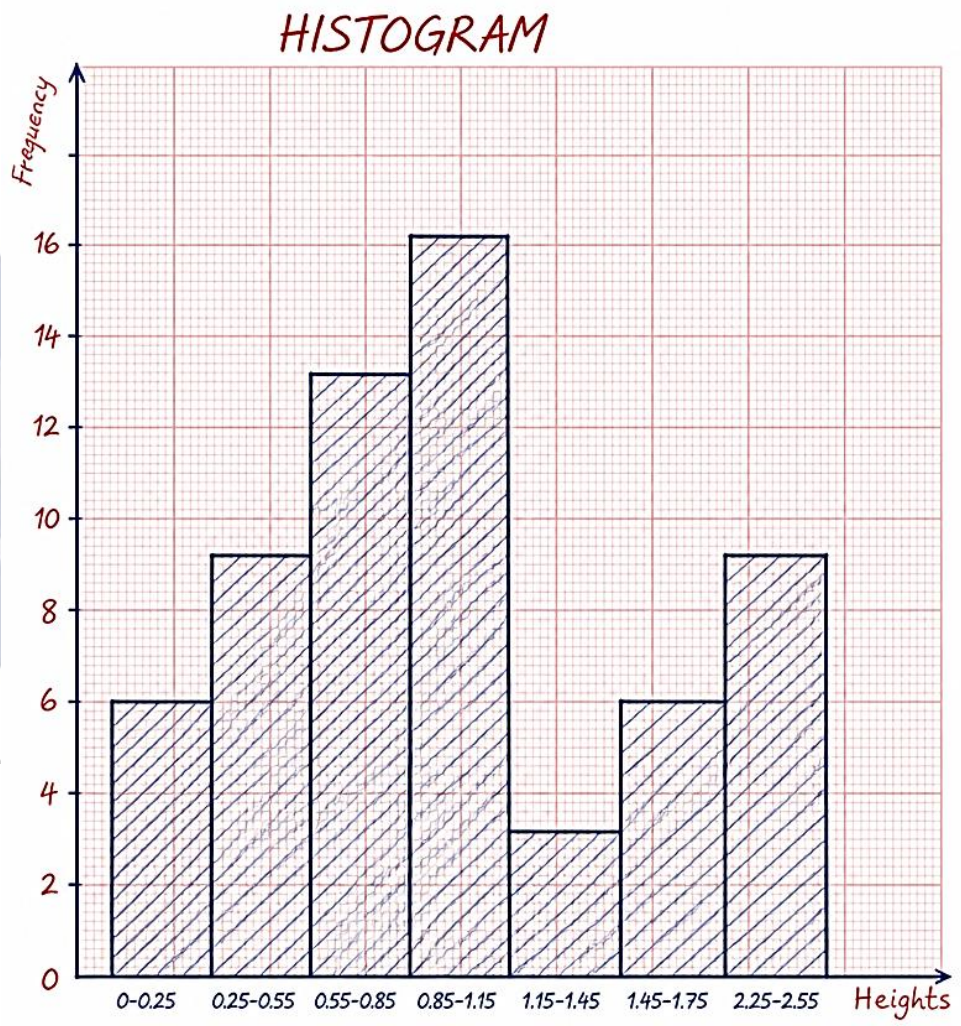
$52 = -8 + 12d$

$12d = 52 + 8 = 60$

$d = 60/12 = 5$

QUESTION 5

(a)



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Height	Class boundary	f	Size	Frequency density
0.1 – 0.3	0.05 – 0.35	6	0.3	20
0.4 – 0.6	0.35 – 0.65	9	0.3	30
0.7 – 0.9	0.65 – 0.95	12	0.3	40
1.0 – 1.4	0.95 – 1.45	15	0.3	30
1.5 – 1.9	1.45 – 1.95	3	0.3	60
2.0 – 2.2	1.95 – 2.25	6	0.3	20
2.3 – 2.5	2.25 – 2.55	9	0.3	30

Histogram: SEE DIAGRAM

(b) Modal height = 1.01 cm

QUESTION 6

Boys = 6, Girls = 8, Total = 14 students

Selecting more girls than boys (5 students):

Either 3 girls and 2 boys, or 4 girls and 1 boy, or 5 girls.

Pr(more girls than boys) = $[{}^{8}C_{3} \times {}^{6}C_{2} + {}^{8}C_{4} \times {}^{6}C_{1} + {}^{8}C_{5}]/{}^{14}C_{5}$

${}^{8}C_{3} = (8 \times 7 \times 6)/(3 \times 2 \times 1) = 56$

${}^{6}C_{2} = (6 \times 5)/(2 \times 1) = 15$

${}^{8}C_{4} = (8 \times 7 \times 6 \times 5)/(4 \times 3 \times 2 \times 1) = 70$

${}^{6}C_{1} = 6$

${}^{14}C_{5} = (14 \times 13 \times 12 \times 11 \times 10)/(5 \times 4 \times 3 \times 2 \times 1) = 2002$

${}^{8}C_{5} = (8 \times 7 \times 6 \times 5 \times 4)/(5 \times 4 \times 3 \times 2 \times 1) = 56$

$= (56(15) + 70(6) + 56)/2002$

$= (840 + 420 + 56)/2002$

$= 1316/2002$

Pr(more girls than boys) = 94/143

QUESTION 7

(a) u = 6 m/s; s = 70 m; v = 20 m/s; a = ?

Using $v^2 = u^2 + 2as$

$20^2 = 6^2 + 2 \times a \times 70$

$400 = 36 + 140a$

$400 - 36 = 140a$

$140a = 364$

$a = 364/140 = 2.6\ m/s^2$

(b) Using $v = u + at$

$20 = 6 + 2.6t$

$20 - 6 = 2.6t$

$14 = 2.6t$

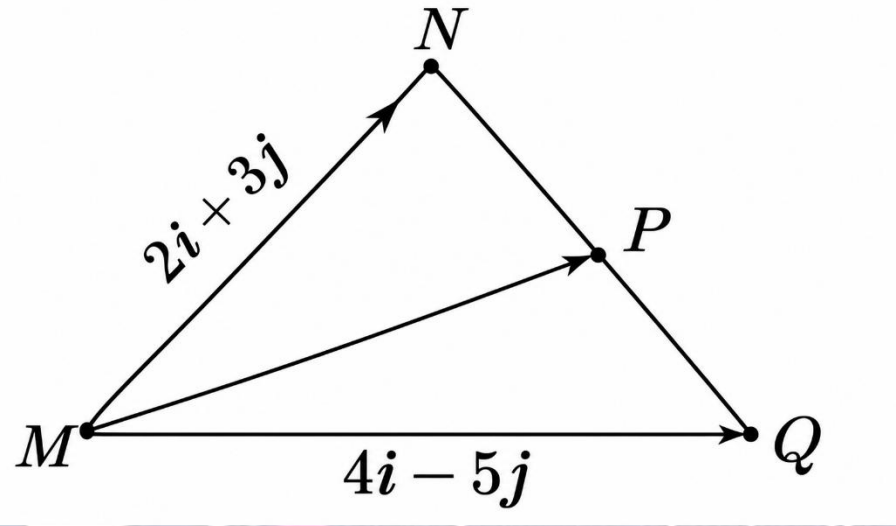
$t = 14/2.6 = 5.384615$

$t \approx 5.38$ seconds

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Hence, time to travel the distance = 5.38 seconds

QUESTION 8



(triangle MNO with P on NO; MN = 8i + 3j and MO = 14i − 5j)

MP = ½(MN + MO)

= ½[(8i + 3j) + (14i − 5j)]

= ½(22i − 2j)

MP = 11i − j

SECTION B

QUESTION 9

(a) $y = 4x - 7/x^2$, differentiating from first principles:

$dy/dx = \lim (\Delta x \to 0) [4(x + \Delta x) - 7/(x + \Delta x)^2 - (4x - 7/x^2)]/\Delta x$

$= \lim [4x + 4\Delta x - 7/(x + \Delta x)^2 - 4x + 7/x^2]/\Delta x$

$= \lim [4\Delta x + 7/x^2 - 7/(x + \Delta x)^2]/\Delta x$

$= \lim [4\Delta x + 7((x + \Delta x)^2 - x^2)/(x^2(x + \Delta x)^2)]/\Delta x$

$= \lim [4\Delta x + 7(2x\Delta x + \Delta x^2)/(x^2(x + \Delta x)^2)]/\Delta x$

Dividing through by Δx:

$= \lim [4 + 7(2x + \Delta x)/(x^2(x + \Delta x)^2)]$

At $\Delta x \to 0$:

$dy/dx = 4 + 7(2x + 0)/(x^2(x + 0)^2)$

$= 4 + 14x/(x^2 \times x^2)$

$= 4 + 14x/x^4$

Hence, $dy/dx = 4 + 14/x^3$

(b) Given $\tan P = 3/(x - 1)$ and $\tan Q = 2/(x + 1)$

$\tan(P - Q) = (\tan P - \tan Q)/(1 + \tan P \tan Q)$

$= [3/(x - 1) - 2/(x + 1)] / [1 + (3/(x - 1))(2/(x + 1))]$

$= [(3(x + 1) - 2(x - 1))/((x - 1)(x + 1))] \div [((x - 1)(x + 1) + 6)/((x - 1)(x + 1))]$

= (3x + 3 − 2x + 2)/(x² − 1 + 6)

tan(P − Q) = (x + 5)/(x² + 5)

QUESTION 10

(a)(i) If 2x + 1 is a factor, then 2x + 1 = 0, so 2x = −1 and x = −½.

g(−½) = 0

For the polynomial g(x) = Mx² + nx + r = 0, at x = −½, substituting:

M(−½)² + n(−½) + r = 0

M/4 − n/2 + r = 0

M − 2n + 4r = 0 ……(1)

(For 2x − 1 = 0, x = 1, as written on the page)

(ii) Hence, g(1) = −6, substituting:

M(1)² + n(1) + r = −6

M + n + r = −6 ……(2)

For x − 2 = 0, x = 2:

g(2) = −5, substituting:

M(2)² + n(2) + r = −5

4M + 2n + r = −5 ……(3)

From equation (2): r = −M − n − 6

From equation (1): M − 2n + 4(−M − n − 6) = 0

M − 2n − 4M − 4n − 24 = 0

−3M − 6n = 24

Dividing through by 3:

M + 2n = −8 ……(4)

From equation (3): 4M + 2n − M − n − 6 = −5

3M + n = 1 ……(5)

Multiply equation (5) by 2:

6M + 2n = 2 ……(6)

Equation (6) minus equation (4):

6M + 2n = 2

M + 2n = −8

5M = 10

M = 2

Substitute M = 2 in equation (5):

3(2) + n = 1

6 + n = 1

n = −5

From equation (2): r = −2 − (−5) − 6 = −2 + 5 − 6

r = −3

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Substituting for M, n and r: $g(x) = 2x^2 - 5x - 3$

(iii) $2x^2 - 5x - 3 = 0$

Factorising: $(2x + 1)(x - 3) = 0$

Either $2x + 1 = 0$, so $x = -½$, or $x - 3 = 0$, so $x = 3$.

The zeros are $-½$ and 3.

(b) $T_3 = {}^8C_2 (x/2)^6 (-1)^2$

$= 8!/((8 - 2)!\, 2!) \times x^6/2^6 \times 1$

$= 8!/(6!\, 2!) \times x^6/64$

$= (8 \times 7 \times 6!)/(6! \times 2 \times 1) \times x^6/64$

$= (7/16)x^6$

QUESTION 11

(a) $(8x^2 + 8x + 9)/((x - 1)(2x + 3)^2) = P/(x - 1) + Q/(2x + 3) + R/(2x + 3)^2$

$8x^2 + 8x + 9 \equiv P(2x + 3)^2 + Q(x - 1)(2x + 3) + R(x - 1)$

Put $x = 1$:

$8(1)^2 + 8(1) + 9 = P(2(1) + 3)^2 + 0 + 0$

$8 + 8 + 9 = P(5)^2$

$25 = 25P$

$P = 1$

Put $x = -3/2$:

$8(-3/2)^2 + 8(-3/2) + 9 = 0 + 0 + R(-3/2 - 1)$

$18 - 12 + 9 = R(-5/2)$

$15 = -5R/2$

$-5R = 30$

$R = -6$

Put $x = 0$:

$8(0)^2 + 8(0) + 9 = P(2(0) + 3)^2 + Q(0 - 1)(2(0) + 3) + R(0 - 1)$

$9 = 9P - 3Q - R$

Substituting for P and R:

$9 = 9(1) - 3Q + 6$

$3Q = 6$

$Q = 2$

Substituting in the original equation:

$(8x^2 + 8x + 9)/((x - 1)(2x + 3)^2) = 1/(x - 1) + 2/(2x + 3) - 6/(2x + 3)^2$

(b) Circumference of a circle $= 2\pi r = 6\pi$

$r = 6\pi/2\pi = 3$

At centre $(-2, 5)$:

$(x + 2)^2 + (y - 5)^2 = 3^2$

$x^2 + 4x + 4 + y^2 - 10y + 25 = 9$

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$x^2 + y^2 + 4x - 10y + 4 + 25 - 9 = 0$

$x^2 + y^2 + 4x - 10y + 20 = 0$

PART II

QUESTION 12

(a)

Class interval	Frequency	Class boundary	Cumulative freq.
11 – 20	5	10.5 – 20.5	5
21 – 30	39	20.5 – 30.5	44
31 – 40	14	30.5 – 40.5	58
41 – 50	40	40.5 – 50.5	98
51 – 60	57	50.5 – 60.5	155
61 – 70	25	60.5 – 70.5	180
71 – 80	11	70.5 – 80.5	191
81 – 90	8	80.5 – 90.5	199
91 – 100	1	90.5 – 100.5	200

(b) Cumulative frequency curve (x-axis: upper class boundary; y-axis: cumulative frequency)



(c)(i) Number of candidates that scored between 24 and 58 (from the graph)

= (143 + 1) − 20

= 124 candidates

(c)(ii) 88th percentile = 88/100 × 200

= 176th mark

= 68.5 marks

QUESTION 13

(a) Green = 4, Non-green = 12

Pr(picking a green ball) = 4/16 = 1/4

Pr(picking a non-green) = 1 − 1/4 = 3/4

Let p = 1/4, q = 3/4

(i) Pr(not picking a green ball) = P(X = 0)

$= {}^5C_0 (1/4)^0(3/4)^5$

$= 1 \times 1 \times 3^5/4^5$

= 243/1024

= 0.2373047

= 0.2373

(ii) Pr(at least 3 green balls)

$= {}^5C_3(1/4)^3(3/4)^2 + {}^5C_4(1/4)^4(3/4)^1 + {}^5C_5(1/4)^5(3/4)^0$

= (10 × 1/64 × 9/16) + (5 × 1/256 × 3/4) + (1 × 1/1024 × 1)

= 90/1024 + 15/1024 + 1/1024

= 106/1024

= 53/512

= 0.1035156

= 0.1035

(b) (The first line is cut off in the photo.)

… = 0

$m^2 + m + 2m - 2 - 1 + 1 - 1 + 2 - 1 - 2 = 0$

$m^2 + 3m - 4 = 0$

Factorising:

$m^2 + 4m - m - 4 = 0$

$m(m + 4) - 1(m + 4) = 0$

$(m - 1)(m + 4) = 0$

Either m − 1 = 0, so m = 1, or m + 4 = 0, so m = −4.

Hence m = 1 or −4

PART III

QUESTION 14

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(a) Initial velocity = 3i + 2j; final velocity = 15i − 4j; time, t = 4 s;

acceleration = a

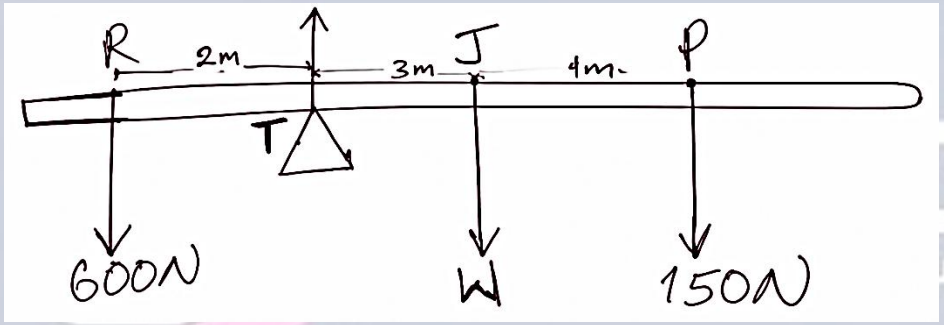
Using v = u + at:

(15i − 4j) = (3i + 2j) + a(4)

(15i − 4j) − (3i + 2j) = 4a

(12i − 2j) = 4a (as written on the page)

a = ¼(12i − 2j)



a = (3i − ½j) m/s²

(b) F = ma

= 2(3i − ½j)

= 6i − j

|F| = √(6² + (−1)²)

= √(36 + 1)

= √37

= 6.0827

|F| = 6.08 N

(c) v = (3i + 2j) + 8(3i − ½j)

= (3i + 2j) + (24i − 4j)

= 27i − 2j

|v| = √(27² + (−2)²)

= √(729 + 4)

= √733

= 27.0739

|v| = 27.07 m/s

QUESTION 15

(a) Let R be Richard, J be John, P be Philip.

(a beam resting on a support T, with 600 N, John's weight W, and 150 N acting at marked distances)

Taking moments about T (clockwise moment = anticlockwise moment):

W × 3 + 150 × 4 = 600 × 2

3W + 600 = 1200

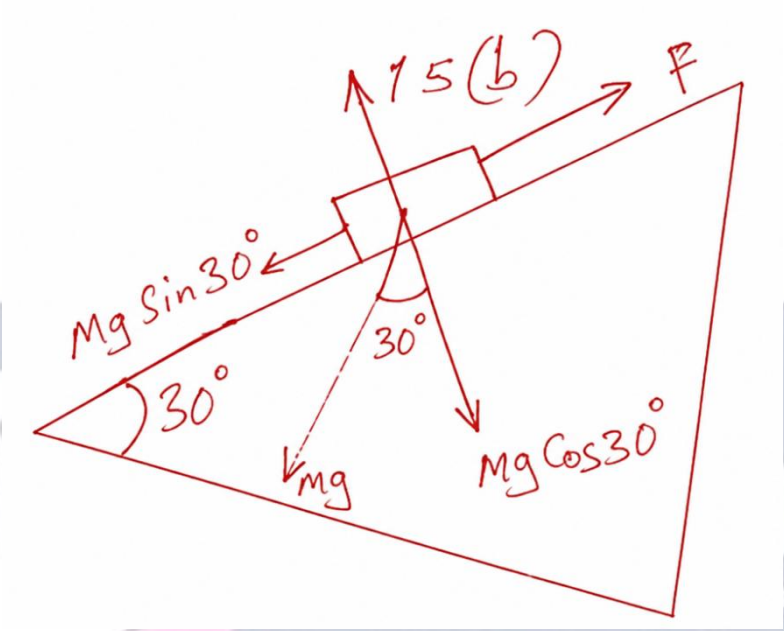
3W = 1200 − 600

3W = 600

W = 600/3 = 200 N

Hence, John's mass = 200/10 = 20 kg

(b)



(a 12 kg block on a plane inclined at 30°, showing mg sin 30°, mg cos 30°, friction F and the force P)

m = 12 kg, weight = 12 × 10 = 120 N, θ = 30°

(i) At the point of moving down:

120 sin 30° + P = F ……(1)

But F = ⅔ × 120 cos 30°

= 80 cos 30°

= 80 × 0.866

= 69.28 N

Substituting for F in equation (1):

120(0.5) + P = 69.28

60 + P = 69.28

P = 69.28 − 60

P = 9.28 N down the plane

(ii) At the point of moving up the plane:

P = 120 sin 30° + F

= 120(0.5) + 69.28

= 60 + 69.28

P = 129.28 N up the plane

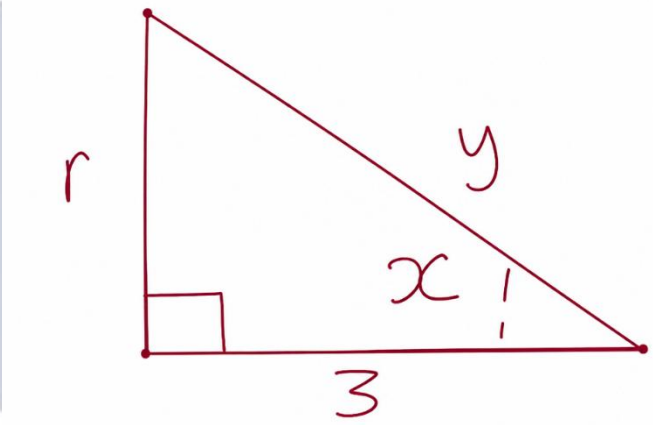
SOLUTIONS TO WASSCE JUNE 2024

FURTHER MATHEMATICS 2 (ESSAY)

SECTION A

QUESTION 1

Given that tan x = 1/3



Let the hypotenuse be y. Applying Pythagoras:

$y^2 = 1^2 + 3^2 = 1 + 9 = 10$

$\Rightarrow y = \sqrt{10}$

Since x lies between 180° and 270° (3rd quadrant):

$\Rightarrow \sin x = -1/\sqrt{10}$ and $\cos x = -3/\sqrt{10}$

From the trigonometric identity:

$\sin 2x = 2 \sin x \cos x$

$= 2(-1/\sqrt{10})(-3/\sqrt{10})$

$= 6/10 = 3/5$

$\Rightarrow (\sin 2x - \cos x)/(2 \tan x + \sin 2x)$

$= [3/5 - (-3/\sqrt{10})] / [2(1/3) + 3/5]$

$= [3/5 + 3/\sqrt{10}] / [2/3 + 3/5]$

$= [(3\sqrt{10} + 15)/(5\sqrt{10})] / [(10 + 9)/15]$

$= (3\sqrt{10} + 15)/(5\sqrt{10}) \times 15/19$

$= (9\sqrt{10} + 45)/(19\sqrt{10})$ (5 cancels into 15, leaving 3)

$= (9\sqrt{10} + 45)/(19\sqrt{10}) \times (19\sqrt{10})/(19\sqrt{10})$

$= [9(19)(10) + 45(19)\sqrt{10}] / [19(19)(10)]$

$= [9(190) + 855\sqrt{10}] / [19(190)]$

$= (1710 + 855\sqrt{10})/3610$

$= 1710/3610 + 855\sqrt{10}/3610$

= 171/361 + (855/3610)√10

= 171/361 + (171/722)√10

QUESTION 2

Sum of natural numbers divisible by 7 between 403 and 603.

a = 406, d = 7, l = 602

$T_n = a + (n - 1)d = l$

406 + (n − 1)7 = 602

7n = 602 − 406 + 7 = 203

n = 29

$S_n = (n/2)(a + l)$

$S_{29} = (29/2)(406 + 602) = (29/2)(1008) = 29 \times 504$

$S_{29} = 14{,}616$

QUESTION 3

∫ from 2.4 to 3.6 of $1/\sqrt{(x^2 - 2)}\,dx$, using x = 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6 with h = 0.2

x	$\sqrt{(x^2 - 2)}$	$1/\sqrt{(x^2 - 2)}$
2.4	1.9391	0.5157
2.6	2.1817	0.4583
2.8	2.4166	0.4138
3.0	2.6458	0.3780
3.2	2.8705	0.3484
3.4	3.0919	0.3234
3.6	3.3106	0.3021

$y_o + y_n = 0.5157 + 0.3021 = 0.8178$

Sum of middle ordinates = 0.4583 + 0.4138 + 0.3780 + 0.3484 + 0.3234

= 1.9219

Trapezium rule: $\int f(x)dx \approx \frac{1}{2}h\{y_o + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\}$

= ½(0.2){0.8178 + 2(1.9219)}

= 0.1(0.8178 + 3.8438)

= 0.1(4.6616)

= 0.46616

QUESTION 4

Equation of the normal to the curve $y = 7x - 5x^2$.

dy/dx = 7 − 10x

At x = 2: gradient of tangent $M_1 = 7 - 10(2) = -13$

Gradient of normal $M_2 = -1/(dy/dx) = 1/13$

At x = 2: $y = 7(2) - 5(2^2) = 14 - 20 = -6$

$(y - y_1)/(x - x_1) = M_2$

(y + 6)/(x − 2) = 1/13

13(y + 6) = x − 2

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13y + 78 = x − 2

13y − x + 80 = 0

QUESTION 5

(a) Given P(M) = 1/2, P(N) = 9/20, P(M ∩ N) = 11/50

P(M) = P(M ∩ N) + P(M ∩ N′)

1/2 = 11/50 + P(M ∩ N′)

P(M ∩ N′) = 1/2 − 11/50 = (25 − 11)/50 = 14/50

P(M ∩ N′) = 7/25

(b) P(M′ ∩ N) = P(N) − P(M ∩ N)

= 9/20 − 11/50

= (45 − 22)/100

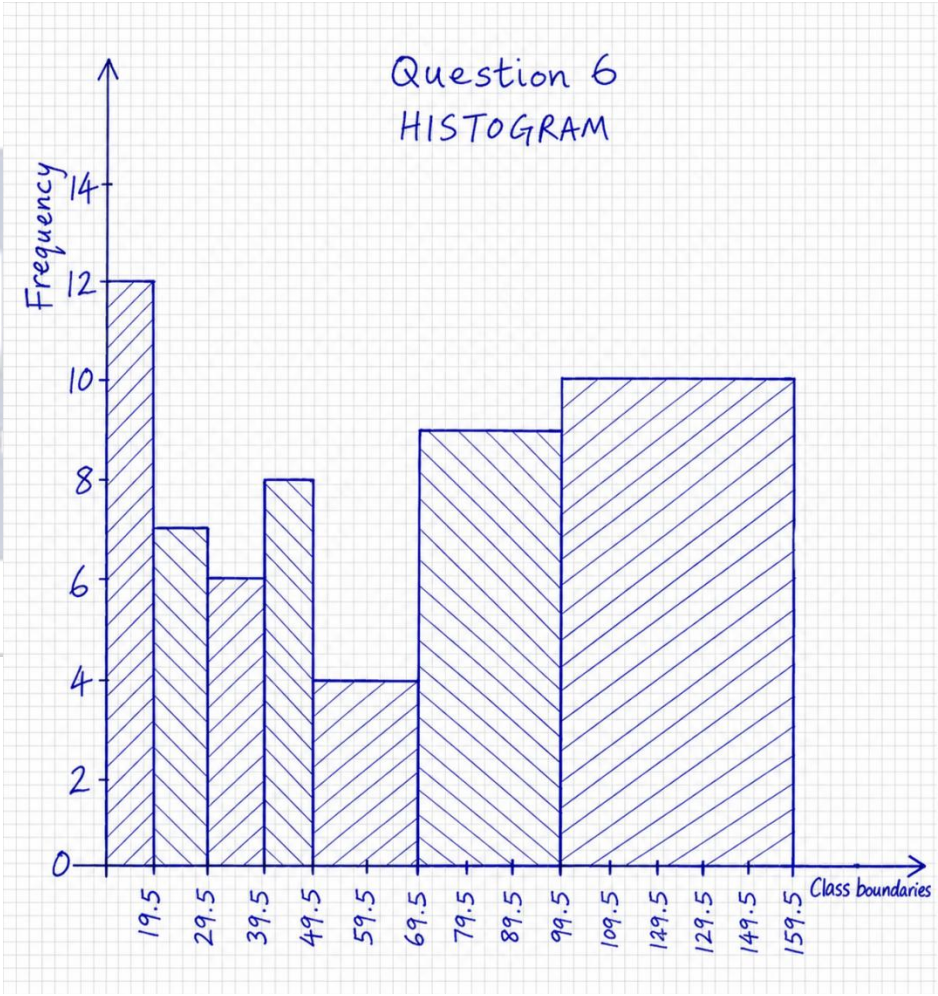
P(M′ ∩ N) = 23/100

QUESTION 6

Distribution of distance (km) against number of villages:

Class interval	Class boundaries	Frequency
0–19	−0.5 to 19.5	12
20–29	19.5 to 29.5	7
30–39	29.5 to 39.5	6
40–49	39.5 to 49.5	8
50–69	49.5 to 69.5	5
70–99	69.5 to 99.5	9
100–149	99.5 to 149.5	10

Histogram:



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QUESTION 7

Given R = 15 N, u = 10 N, v = 12 N. Applying the cosine rule:

$R^2 = u^2 + v^2 + 2uv \cos\theta$

$15^2 = 10^2 + 12^2 + 2 \times 10 \times 12 \cos\theta$

$225 = 244 + 240 \cos\theta$

$240 \cos\theta = -19$

$\cos\theta = -19/240 = -0.07917$

$\theta = \cos^{-1}(-0.07917) = 94.54°$

The angle between them is 94.54°

QUESTION 8

(a)

M = 40 kg, so weight = 40 × 10 = 400 N, θ = 30°

Component of weight down the slope = 400 sin 30° = 400 × 0.5 = 200 N

F_R = 420 − 200 = 220 N

Maximum friction force = 220 N

(b) $F = \mu R = \mu mg \cos\theta$

$\mu = F/R = 220/(400 \cos 30°)$

$= 220/(400 \times 0.866) = 220/346.4$

$= 0.63508$

$\mu \approx 0.6351$

SECTION B

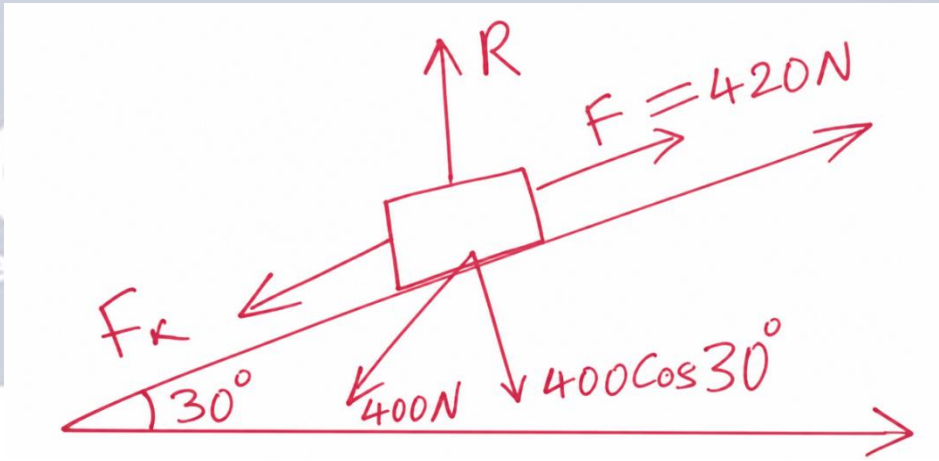
PART I

QUESTION 9

$y = 8x + 27/(2x^2) = 8x + (27/2)x^{-2}$

(a) $dy/dx = 8 + (27/2)(-2)x^{-3}$

$dy/dx = 8 - 27/x^3$



(b) For stationary points, dy/dx = 0:

$8 - 27/x^3 = 0$

$8x^3 = 27$

$x^3 = 27/8$

$x = 3/2$

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At $x = 3/2$: $y = 8(3/2) + 27/(2(3/2)^2) = 12 + 27/(9/2) = 12 + 6 = 18$

Stationary point = $(3/2, 18)$

(c) The stationary point is the minimum gradient at (2, 2).

$dy/dx = 8 - 27/x^3$

At (2, 2): $dy/dx = 8 - 27/8 = (64 - 27)/8 = 37/8$

Gradient of normal = $-1/(dy/dx) = -8/37$

Equation of the normal at (2, 2):

$(y - 2)/(x - 2) = -8/37$

$37(y - 2) = -8(x - 2)$

$37y - 74 = -8x + 16$

$37y + 8x - 90 = 0$

QUESTION 10

$P = [-4\ -1; 2\ 0]$, $Q = [0\ 1; 6\ -9]$, $R = [1\ -2; 3\ 5]$

(Matrices are written row by row, with a semicolon between rows.)

(i) $2P - 3R + Q = [-8\ -2; 4\ 0] - [3\ -6; 9\ 15] + [0\ 1; 6\ -9]$

$= [-11\ 5; 1\ -24]$

(ii) $QR = [0\ 1; 6\ -9][1\ -2; 3\ 5]$

$= [0 + 3, 0 + 5; 6 - 27, -12 - 45]$

$= [3\ 5; -21\ -57]$

(iii) Inverse of R:

$|R| = (1 \times 5) - (-2 \times 3) = 5 + 6 = 11$

$R^{-1} = (1/11)[5\ 2; -3\ 1]$, or $[5/11\ 2/11; -3/11\ 1/11]$

QUESTION 11

(a) $9x/((2x + 1)(x^2 + 1)) = A/(2x + 1) + (Bx + C)/(x^2 + 1)$

$9x = A(x^2 + 1) + (Bx + C)(2x + 1)$

$9x = Ax^2 + A + 2Bx^2 + Bx + 2Cx + C$

Comparing coefficients:

$B + 2C = 9$ …(1)

$A + 2B = 0$ …(2)

$A + C = 0$ …(3)

From (2) and (3): $2B - C = 0$

Multiplying (1) by 2: $2B + 4C = 18$

Subtracting: $5C = 18$, so $C = 18/5$

$A + C = 0$, so $A = -18/5$

$B = 9 - 2C = 9 - 36/5 = 9/5$

$9x/((2x + 1)(x^2 + 1)) = -18/(5(2x + 1)) + (9x + 18)/(5(x^2 + 1))$

(b) ${}^{2m}P_2 - 10 = {}^{m}P_2$

$(2m)!/(2m - 2)! - 10 = m!/(m - 2)!$

$2m(2m - 1) - 10 = m(m - 1)$

$4m^2 - 2m - 10 = m^2 - m$

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$3m^2 - m - 10 = 0$

$(3m + 5)(m - 2) = 0$

$m = -5/3$ or $m = 2$

Since m must be positive, $m = 2$

PART II

QUESTION 12

(a) The data in order are 11, 12, $(2x + y)$, $(x + 2y)$, 14, $(y^2 - 2x)$, with median 13½.

Median $= ((2x + y) + (x + 2y))/2 = 27/2$

$3x + 3y = 27$

$x + y = 9$ …(1)

Recall $x + 1 = y$, so $x - y = -1$ …(2)

Substituting $y = x + 1$ in (1): $2x + 1 = 9$, so $2x = 8$

$x = 4$, $y = 5$

(b) Substituting for x and y in the original values:

11, 12, $(2\times4 + 5)$, $(4 + 2\times5)$, 14, $(5^2 - 2\times4) = 11, 12, 13, 14, 14, 17$

Mean $\bar{x} = (11 + 12 + 13 + 14 + 14 + 17)/6 = 81/6 = 13.5$

x	**$x - \bar{x}$**	**$(x - \bar{x})^2$**
11	−2.5	6.25
12	−1.5	2.25
13	−0.5	0.25
14	0.5	0.25
14	0.5	0.25
17	3.5	12.25
	Σ =	21.5

Standard deviation $= \sqrt{(\Sigma(x - \bar{x})^2/n)} = \sqrt{(21.5/6)} = \sqrt{3.5833}$

S.D. = 1.89

QUESTION 13

(a) Pass = 70/100 = 7/10, Fail = 30/100 = 3/10, number of selections = 12

$P(\text{at least } 2) = P(x \geq 2) = 1 - \{P(x = 0) + P(x = 1)\}$

$P(x = 0) = {}^{12}C_{12}\,(7/10)^{12}\,(3/10)^0 = 1 \times 0.0138412 \times 1 = 0.0138412$

$P(x = 1) = {}^{12}C_{11}\,(7/10)^{11}\,(3/10)^1 = 12 \times 0.0197736 \times 0.3 = 0.07118496 \approx 0.0711850$

$P(x \geq 2) = 1 - (0.0138412 + 0.0711850)$

$= 1 - 0.0850262$

$= 0.9149738$

(b) Exactly half of them = 6:

$P(x = 6) = {}^{12}C_6\,(7/10)^6\,(3/10)^6$

$= 924 \times 0.117649 \times 0.000729$

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= 0.07924795

≈ 0.0792479

(c) Not more than 2, i.e. $x \leq 2$:

$P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2)$

$= (1 \times 0.0138412 \times 1) + (12 \times 0.0197736 \times 0.3) + (66 \times 0.0282475 \times 0.09)$

$= 0.0138412 + 0.07118496 + 0.1677902$

$= 0.25281636$

PART III

QUESTION 14

(a) Resolving each force into i and j components:

$F_1 = 10 \sin 90° \, i + 10 \cos 90° \, j = 10i + 0j$, so $F_1 = (10, 0)$

$F_2 = 16 \sin 180° \, i + 16 \cos 180° \, j = 0i - 16j$, so $F_2 = (0, -16)$

$F_3 = 7 \sin 30° \, i + 7 \cos 30° \, j = 3.5i + 6.062j$, so $F_3 = (3.5, 6.062)$

$F_4 = 12 \sin 300° \, i + 12 \cos 300° \, j = -10.39i + 6j$, so $F_4 = (-10.39, 6)$

(b)(i) $F = (10, 0) + (0, -16) + (3.5, 6.062) + (-10.39, 6) = (3.11, -3.938)$

Magnitude of resultant force $R = \sqrt{((3.11)^2 + (-3.938)^2)}$

$= \sqrt{(9.6721 + 15.507844)}$

$= \sqrt{25.179944}$

= 5.01796 N

R ≈ 5.02 N

(ii) F = ma

5.02 = 1.2a

a = 5.02/1.2 = 4.182

$a \approx 4.18 \text{ m/s}^2$

QUESTION 15

(a) Displacement: $20x = 4t^2 + t^3$, so $x = t^2/5 + t^3/20$

(i) Velocity = $dx/dt = 2t/5 + 3t^2/20$

When t = 15 s:

$V = (2/5)(15) + (3/20)(15^2) = 6 + 675/20 = 6 + 33.75$

V = 39.75 m/s

(ii) Acceleration = $dv/dt = 2/5 + 6t/20$

Initial acceleration (t = 0): a = 2/5 + (6/20)(0)

$a = 2/5 \text{ m/s}^2$

(iii) When $a_1 = 8a_o$:

$8(2/5) = 2/5 + 6t/20$

$16/5 = (8 + 6t)/20$

$20(16) = 5(8 + 6t)$

$320 = 40 + 30t$

$30t = 280$

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t = 280/30 = 9⅓ = 9.33

t = 9.33 seconds

(b) Given M_1 = 6 kg, body 1 has u_1 = 7 m/s; M_2 = ?, body 2 has u_2 = −5 m/s.

From the conservation of linear momentum:

$M_1u_1 + M_2u_2 = (M_1 + M_2)V$

$6(7) + M_2(-5) = (6 + M_2)4$

$42 - 5M_2 = 24 + 4M_2$

$42 - 24 = 4M_2 + 5M_2$

$18 = 9M_2$

M_2 = 2 kg

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